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A fuzzy extension of MEREC method using parabolic measure and its applications

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Abstract

Human qualitative judgments are often characterized by uncertainty and predictability. Decision-makers tend to be more confident in making linguistic decisions than in crisp value judgments. MEREC is capable of achieving relative objective weights of several conflicting criteria. This paper contains two parts, first, the extension of MEREC method in fuzzy circumstances based on linguistic terms in which a parabolic measure has been used to calculate the overall performance of alternatives as it is able to work according the definition of TFNs and to show the applicability, a simple decision matrix is analysed in a fuzzy environment. Second, a new hybrid ranking methodology Fuzzy MEREC-TOPSIS for multi criteria decision making. Further, to illustrate the credibility and effectiveness of the proposed hybrid ranking method, a real-world example of stock selection has been used. The portfolio is constructed using ranking of the stocks received through the proposed method and capital is allocated according to the order of preference of stocks. To validate the proposed ranking model, the next 30 days closing price of each stock is predicted by a deep recurrent neural network and the portfolio for future investments is analysed. The results of the future analysis validate the credibility of the portfolio.

Keywords: MEREC, Fuzzy MEREC, Fuzzy TOPSIS, Stock portfolio selection, MCDM.

1. Introduction

Real-world problems often involve making complex decisions with many conflicting objectives. Multi-criteria decision making (MCDM) is a major part of operational research. Decision makers may choose the optimal one from a set of options rest on some stipulated criteria (Plous, 1993; Janis and Mann, 1977). Decision making is called multi-criteria decision making when several criteria are contemplated together to arrive at a conclusion (Triantaphyllou, 2013; Zeleny and Cochrane, 1973). The main problem is how to enumerate a set of alternatives rest on multiple conflicting criteria. Multi-criteria decision-making format is a controlled decision tool for calculating the weight of the evaluation criterion as well as ranking the alternatives present in problems with quantitative and qualitative criteria.

MCDM problems could be classified into two classes: Multiple-Objective Decision Making (MODM) for designing the best solution and Multiple-Attribute Decision-Making (MADM) for choosing the best alternative. MODM methods refer to handling continuous problems with infinite number of options. On the other hand, MADM methods refer to discrete representations of a problem with many conflicting criteria and a limited number of alternatives. MCDM is generally used to represents the discrete MADM. Over the past decades, several MCDM approaches have been developed by researchers such as ELECTRE (Elimination and Choice Expressing Reality) (Benayoun et al., 1996), TOPSIS (Technique for order preference by similarity to an ideal solution) (Hwang and Yoon, 1981; Yoon, 1987), VIKOR (Duckstein and Opricovic, 1980), COPRAS (Complex Proportional Assessment) (Zavadskas, 1994; Zavadskas et al., 2001), SWARA (step-wise weight assessment ratio analysis) (Keršuliene et al., 2010), AHP (Analytic Hierarchy Process) (Saaty, 1980), ANP (Analytic Network Process) (Saaty, 1996; Saaty, 2005), BWM (Best Worst Method) (Rezaei, 2015; Rezaei, 2016), BCM (Base-Criterion Method) (Haseli et al., 2019), CRITIC (CRiteria Importance Through Inter-criteria Correlation) (Diakoulaki et al., 1995) and MEREC (MEthod based on the Removal Effects of Criteria) (Ghorabaee et al., 2021).

There are two main goals for solving practical problems by MCDM methods: (a) calculating the optimum weight of the criterion; (b) setting the rank of the alternatives. Criteria can be considered an important information source during the process of decision-making issues. How we receive the criteria weights is one of the most epochal and intricate process in handling of MCDM problems. The criterion weight reflects their importance and the final evaluation results highly depend on the criterion weights. The methods of obtaining criterion weights falls into two categories: subjective and objective. Subjective weights are determined based on the decision maker's level of preference. Expressing preferences is a mental task, and an increase in the number of criteria somewhere reduces the accuracy of the DM’s preferences. On the other hand, Objective weights are determined using a conspicuous computational procedure rest on the pilot data.

Recently, a new method called MEREC has been proposed by Ghorabaee et al. (2021) to compute the objective weighting of the criterion in the MCDM issues. The method is rest on the expulsion impact of each criterion on the collective performance of the alternatives for computing the criterion weighting. Criteria that have a greater impact on performance are given more importance. Causality is the basic idea behind this approach. In this method, a logarithmic function is used to calculate the aggregate performance of the alternatives.

In MCDM, uncertainty and incompleteness are other important issues that typically emerge with vague, incomplete, subjective and conflicting data. Due to the increasing complexity of decision-making environments, fuzzy sets are commonly used by decision makers to express their impenetrable and uncertain preference information (Zadeh, 1965). Many fuzzy-based MCDM techniques have been developed such as fuzzy TOPSIS (Zhang and Xu, 2015; Guo and Zhao, 2015), fuzzy ELECTRE (Chen and Xu, 2015) and fuzzy BWM (Guo and Zhao, 2017), fuzzy BCM which have been used in many practical problems, such as situation assessments (Lu et al., 2008), weapon selection for defense systems (Degdeviren et al., 2009), stock portfolio selection (Chen and Hung, 2009; Narang et al., 2021; Narang et al., 2022) and supplier selection under sustainability (Rao et al., 2017).

TOPSIS method was first developed in 1981 by Hwang and Yoon (1981) to select the best alternative from the set of alternatives. The underlying principle of the TOPSIS method is that the selected option’s should have minimum distance from the positive ideal solution and a maximum distance from the negative ideal solution. Chen (2000) developed the TOPSIS method for decision-making problems in fuzzy environment. It is a reliable MCDM technique to reveal decision information under a fuzzy environment. Fuzzy TOPSIS has been used in many practical issues such as supplier evaluation and selection in supply chain management (Chen et al., 2006), optimal site selection of electric vehicle charging station (Guo and Zhao, 2015) and spill way selection (Balioti et al., 2018), etc.

In the present work, firstly, the MEREC method is developed in ambiguous environment. Second, a new ranking methodology has been developed by integrating two approaches Fuzzy MEREC and Fuzzy TOPSIS. The applicability and functionality of the proposed hybrid ranking method is determined by applying the method to the stock portfolio selection process. A new ranking hybrid methodology fuzzy MEREC-TOPSIS (a) provides a coherent and precise approach to solving complex multi-criteria decision-making problems despite uncertainty; (b) is less computational.

We systematize the rest of paper as follows: Section 2 preliminaries; Section 3 gives the extension of the MEREC method to the fuzzy environment; Section 4 introduces the proposed hybrid ranking methodology and its features; Section 5 explores the results; Section 6 shows findings and conclusion.

2. Materials & Methods- Fuzzy MEREC-TOPSIS

2.1 Preliminaries

Definition 2.1 (Carlsson and Fuller, 2001) A fuzzy number $\tilde{a}$ on R is defined as a triangular fuzzy number (TFNs) if its membership function $\mu_{\tilde{a}}(x): R \to [0,1]$ is equal to

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < p \\ \frac{x-p}{q-p}, & p \le x \le q \\ \frac{r-x}{r-q} & q \le x \le r \\ 0, & x > r \end{cases} \quad (1)$$

where p, q and r are respectively represents the lower, middle and upper values of the support of $\tilde{a}$, all of which are crisp values ($-\infty < p \le q \le r < \infty$). A TFN can be shown as a triplet (p, q, r).

Definition 2.2 (Zhao and Guo, 2014) The graded mean integration representation (GMIR) $R(\tilde{a})$ of a TFN $\tilde{a}$ represent the ranking of TFN. Let $\tilde{a}_i = (p_i, q_i, r_i)$, then

$$(\tilde{a}_i) = \frac{p_i + 4q_i + r_i}{6} \quad (2)$$

2.2 Fuzzy MEREC

Human qualitative judgments sometimes bear a characteristic of ambiguity and approximate. Decision-makers give their opinions through linguistic terms. Fuzzy set is a tool to quantitatively characterize the mental and linguistic opinions of decision-makers. So, fuzzy information recruitment may be a better approach to extend MEREC (Ghorabaee et al., 2021) in ambiguous circumstances to overcome many multi-criteria decision-making problems.

In the developed methodology, decision makers express their opinion in linguistic terms based on the initial data such as extremely high, moderately low, strongly low etc., which can be converted into their respective TFNs to form a decision matrix. Parabolic measure, a U-shaped non-linear increasing function (Figure 1), has been introduced to calculate the overall performance of the criterion. Parabolic measure is easy to apply and able to work according to the definition of TFNs. The difference between the collective performance of the alternatives and the performance of these after deleting the criterion is calculated by using the "Euclidean distance measure".

The steps of fuzzy MEREC method for deriving the objective weights of the criteria are as follows:

Step 1. Identification of criteria and alternatives.

In the first step, a set of criteria $\{c_1, c_2, c_3, \ldots, c_n\}$ and alternatives $\{a_1, a_2, a_3, \ldots, a_m\}$ are determined and evaluated according to the opinion of the decision makers. Also, identify the cost-criteria (C) and benefit-criteria (B).

Step 2. Determination of the decision matrix.

A fuzzy decision matrix is created according to the opinion of the decision makers based on the initial data which reflects the ratings or values of each alternative related to each criterion. The decision-makers give their opinions through linguistic terms. TFNs are listed in Table 1 to denote the linguistic sets used by decision maker. Assuming the performance rating (according to benefit and cost criteria) assigned by the decision maker to the i^{th} alternative in relation to the j^{th} criterion is $x_{ij} = \left(p_{ij}, q_{ij}, r_{ij}\right)$.

A fuzzy decision matrix is as follows:

$$D = \begin{pmatrix} (p_{11}, q_{11}, r_{11}) & (p_{12}, q_{12}, r_{12}) & \dots & (p_{1n}, q_{1n}, r_{1n}) \\ (p_{21}, q_{21}, r_{21}) & (p_{22}, q_{22}, r_{22}) & \dots & (p_{2n}, q_{2n}, r_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (p_{m1}, q_{m1}, r_{m1}) & (p_{m2}, q_{m2}, r_{m2}) & \dots & (p_{mn}, q_{mn}, r_{mn}) \end{pmatrix} \tag{3}$$

Table 1. Linguistic expressions and their corresponding TFNs for assessment

Linguistic expressions	Abbreviation	TFNs	0.1-0.9 scale
Extremely Low	EL	(0.1,0.1,0.2)	0.1
Very Strongly low	VSL	(0.1,0.2,0.3)	0.2
Strongly low	SL	(0.2,0.3,0.4)	0.3
Moderately low	ML	(0.3,0.4,0.5)	0.4
Moderately high	MH	(0.4,0.5,0.6)	0.5
Very high	VH	(0.5,0.6,0.7)	0.6
Strongly high	SH	(0.6,0.7,0.8)	0.7
Very Strongly high	VSH	(0.7,0.8,0.9)	0.8
Extremely high	EH	(0.8,0.9,0.9)	0.9

Step 3. Normalization of a fuzzy decision matrix.

The normalized fuzzy decision matrix $\widetilde{D}_{ij} = \left[\tilde{d}_{ij}\right]_{m\times n}$ can be constructed in the following way (Huang et al., 2013; Gani, 2012):

$$\tilde{d}_{ij} = \frac{x_{ij}-\min_i(x_{ij})}{\max_i(x_{ij})-\min_i(x_{ij})} = \left(d_{ij}^{(p)}, d_{ij}^{(q)}, d_{ij}^{(r)}\right),\ \ i = 1,2,,3 \dots., m \tag{4}$$

Step 4. Overall performance of the alternatives.

A parabolic measure is employed to obtain the overall performance of the alternatives. It is a non-linear increasing function (Figure 1) and is able to work according to the definition of TFNs.

The following equation is applied for the calculation.

$$\tilde{P}_i = \frac{1}{n}\sum_{j=1}^{n}\left(\tilde{d}_{ij}\right)^2$$

$$\tilde{P}_i = \left(\frac{1}{n}\sum_{j=1}^{n}\left(d_{ij}^{(p)}\right)^2, \frac{1}{n}\sum_{j=1}^{n}\left(d_{ij}^{(q)}\right)^2, \frac{1}{n}\sum_{j=1}^{n}\left(d_{ij}^{(r)}\right)^2\right) = (\tilde{p}_i, \tilde{q}_i, \tilde{r}_i) \tag{5}$$

Step 5. Enumeration of the performance of alternatives by deleting each criterion.

The alternative's performance is calculated on the basis of deletion of every criterion separately. For this, parabolic measure is used as in the prior step. The collective performance of i^{th} alternative concerning the deletion of j^{th} criterion is calculated as follows.

$$\tilde{P}'_{ij} = \left(\frac{1}{n}\sum_{k,k\neq j}\left(d_{ih}^{(p)}\right)^2, \frac{1}{n}\sum_{k,k\neq j}\left(d_{ih}^{(q)}\right)^2, \frac{1}{n}\sum_{k,k\neq j}\left(d_{ih}^{(r)}\right)^2\right) = \left(\tilde{p}'_{ij}, \tilde{q}'_{ij}, \tilde{r}'_{ij}\right) \tag{6}$$

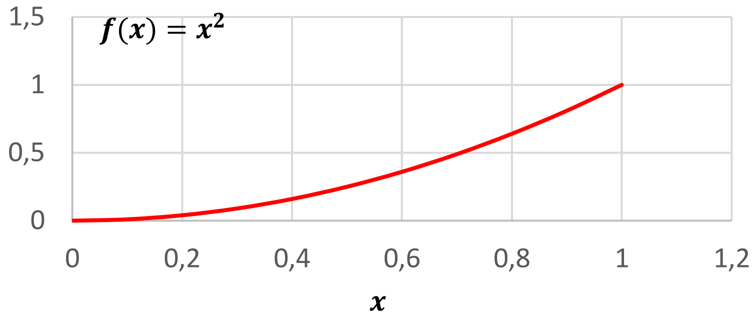

Figure 1. Parabolic measure

Step 6. Computation of the summation of Euclidean distances.

The expulsion impact of the j^{th} criterion on the basis of the values received from step 3 and step 4 is calculated as follows:

$$\tilde{E}_j = \left(\sqrt{\frac{1}{m}\sum_i (\tilde{p}'_{ij} - \tilde{p}_i)^2}\ , \sqrt{\frac{1}{m}\sum_i (\tilde{q}'_{ij} - \tilde{q}_i)^2}\ , \sqrt{\frac{1}{m}\sum_i (\tilde{r}'_{ij} - \tilde{r}_i)^2} \right) \quad (7)$$

Step 7. Assessment of final weights of the criteria.

The objective weight of each criterion is calculated using the following equation:

$$\widetilde{w}_j = \frac{\tilde{E}_j}{\sum_k \tilde{E}_k} \quad (8)$$

After solving Eq.8, the objective weights can be transferred into crisp numbers by applying GMIR method (Eq.2), if needed.

2.2.1 Illustrative example

In this sub-section, a simple example is adopted from (Ghorabaee et al., 2021) and tackled by MEREC in fuzzy environment. Table 2 shows the initial data. To obtain a fuzzy decision matrix, assessment values given by a decision maker based on Table 1 and Table 2 is mentioned in Table 3. The normalized decision matrix (Table 4) is constructed by make use of Eq. 4. The overall performance (Table 5) of alternatives is calculated by using Eq. 5.

Table 2. Initial data

Alternatives/Criteria	c_1	c_2	c_3	c_4
a_1	450	8000	54	145
a_2	10	9100	2	160
a_3	100	8200	31	153
a_4	220	9300	1	162
a_5	5	8400	23	158

Table 3. Assessment values given by decision maker based on initial data

Alternatives/Criteria	c_1	c_2	c_3	c_4
a_1	EH	EL	EL	ML
a_2	SL	SL	VSH	VSL
a_3	SH	EL	SL	SL
a_4	VH	MH	EH	EL
a_5	EL	EL	ML	EL

Table 4. Normalized fuzzy decision matrix

Alternatives/Criteria	c_1	c_2	c_3	c_4
a_1	(1,1,1)	(0,0,0)	(0,0,0)	(1,1,1)
a_2	(0.14,0.25,0.29)	(0.25,0.50,0.67)	(0.75,0.88,1.00)	(0,0,0)
a_3	(0.71,0.75,0.86)	(0,0,0)	(0.13,0.25,0.29)	(0.00,0.33,0.50)
a_4	(0.86,0.88,1.00)	(1,1,1)	(0.88,1.00,1.00)	(0,0,0)
a_5	(0,0,0)	(0,0,0)	(0.25,0.38,0.43)	(0,0,0)

Table 5. Collective performance of alternatives

Alternatives	$\tilde{P}_i$
a_1	(0.500,0.500,0.500)
a_2	(0.161,0.270,0.382)
a_3	(0.131,0.184,0.267)
a_4	(0.625,0.691,0.750)
a_5	(0.016,0.035,0.046)

The performance of alternatives (Table 6) after removing each criterion is determined using Eq. (6). The computation of the summation of Euclidean distances is performed using Eq. 7 and the calculated values are mentioned in Table 7. Finally, the objective weights of each criterion are calculated using Eq. (8).

Table 6. Values of $\tilde{P}'_{ij}$

Alternatives/Criteria	c_1	c_2	c_3	c_4
a_1	(0.250,0.250,0.250)	(0.500,0.500,0.500)	(0.500,0.500,0.500)	(0.250,0.250,0.250)
a_2	(0.156,0.254,0.361)	(0.146,0.207,0.270)	(0.021,0.078,0.132)	(0.161,0.270,0.382)
a_3	(0.004,0.043,0.083)	(0.131,0.184,0.267)	(0.128,0.168,0.246)	(0.131,0.156,0.204)
a_4	(0.441,0.500,0.500)	(0.375,0.441,0.500)	(0.434,0.441,0.500)	(0.625,0.691,0.750)
a_5	(0.016,0.035,0.046)	(0.016,0.035,0.046)	(0.000,0.000,0.000)	(0.016,0.035,0.046)

Table 7. Values of $\tilde{E}_j$

Alternatives/Criteria	c_1	c_2	c_3	c_4
$\tilde{E}_j$	(0.150,0.154,0.178)	(0.112,0.115,0.122)	(0.106,0.142,0.160)	(0.112,0.112,0.115)

$\widetilde{w}_1 = (0.261,0.295,0.371)$; $\widetilde{w}_2 = (0.195,0.220,0.255)$; $\widetilde{w}_3 = (0.185,0.271,0.333)$;
$\widetilde{w}_4 = (0.194,0.215,0.240)$;

By Make use of Eq. 2, the obtained objective weights can be converted to crisp values which are
$\widetilde{w}_1 = 0.302$; $\widetilde{w}_2 = 0.222$; $\widetilde{w}_3 = 0.267$; $\widetilde{w}_4 = 0.215$;

2.2.2 Comparison analysis

The proposed model is straightforward and less calculative as a simple normalization technique has been used in which the higher rating is given to the maximum value of the criterion and lower rating is given to the minimum value of the criterion under fuzzy environment. Use of Parabolic measure makes the proposed model more efficient because parabolic measure is an increasing function so there is no need to keep in mind that the smaller value of the normalized matrix yields a greater value of performances. Also, parabolic measure is more capable to work with the TFNs as compare to logarithmic measure. Euclidean measure has been used to calculate the distance between the performance of the alternatives instead of absolute deviation. The advantage of this measure is that the distance between any two alternatives is not affected by the addition of new alternatives to the analysis. Squaring makes the algebra much easier to work with and offers properties that the absolute method does not.

In MEREC (Ghorabaee et al., 2021) based on the initial data, lower rating (in crisp value) is given to the maximum value of the criterion and higher rating (in crisp value) is given to the minimum value of the criterion. Thereafter, a logarithmic measure is used to calculate the overall performance of the criterion, taking into account that the smaller value of the normalized matrix yields a greater value of performances. This makes the model (Ghorabaee et al., 2021) little bit complicated.

Also, the order of precedence of objective weights obtained using the proposed method (Extension of MEREC in ambiguous circumstances) is the same as the order of precedence of objective weights obtained by MEREC (Ghorabaee et al., 2021) . Table 8 shows that the fuzzy MEREC is fully capable of considering the ambiguity of the decision maker in the decision-making process.

Table 8. Comparison of MEREC and the proposed Fuzzy MEREC

Methods	$\boldsymbol{w_1}$	$\boldsymbol{w_2}$	$\boldsymbol{w_3}$	$\boldsymbol{w_4}$
MEREC (Ghorabaee et al.,2021)	0.575	0.014	0.401	0.009
Proposed method (Fuzzy MEREC)	0.302	0.222	0.267	0.215

2.3 A new hybrid ranking approach "Fuzzy MEREC-TOPSIS" for multi-criteria decision making

In this section, the second part of the proposed methodology, the fuzzy MEREC-TOPSIS method, is introduced.

A Decision-making problem is selected which includes a set of m alternatives $\{a_1, a_2, \ldots., a_m\}$ and a set of n criteria $\{C_1, C_2, \ldots., C_n\}$. Decision makers identify cost-criteria (C') and benefit-criteria (B') for the decision-making problem based on their knowledge and experience. The assessment value assigned by decision maker to the i^{th} alternative with respect to the j^{th} criteria as per the initial data and Table 1 is $x_{ij} = \left(p_{ij}, q_{ij}, r_{ij}\right)$.

The steps of fuzzy MEREC-TOPSIS method to investigate the importance and priority of the alternatives are as follows.

Step 1. Determine a fuzzy decision matrix $D = \left[x_{ij}\right]_{m\times n}$.

Step 2. The normalized fuzzy decision matrix $\widetilde{D}_{ij} = \left[\tilde{d}_{ij}\right]_{m\times n}$ can be constructed using Eq.4.

Step 3. The objective weights of the criteria can be calculated by using fuzzy MEREC (Eq. 5,6,7 and 8).

Step 4. The weighted normalized fuzzy decision matrix is defined by $V = \left[v_{ij}\right]_{m\times n}$ where

$$v_{ij} = \tilde{d}_{ij} \otimes \widetilde{w}_j \quad i = 1,2,..,m, \ j = 1,2,\ldots,n \tag{9}$$

Step 5. fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) might be described as follows.

$$A'^{+} = (v_1^+, v_2^+, \ldots, v_n^+) \text{ where } v_j^+ = \max_i v_{ij},$$
$$A'^{-} = (v_1^-, v_2^-, \ldots, v_n^-) \text{ where } v_j^- = \min_i v_{ij}; \tag{10}$$

Step 6. Calculate the Euclidean distance of each alternative from FPIS and FNIS, respectively.

$$s_i^+ = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^+\right)^2} \qquad i = 1,2,\ldots,m,$$
$$s_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2} \qquad i = 1,2,\ldots,m; \tag{11}$$

Step 7. The closeness coefficient that is defined to determine the ranking of alternatives can be calculated as follows.

$$CC_i = \frac{s_i^-}{s_i^- + s_i^+}, \quad i = 1,2,\ldots,m. \tag{12}$$

The value of the closeness coefficient varies between 0 to 1. The larger the value of the closeness coefficient, the greater the priority of the alternative.

3. Results and a case study

In this section, the application of our proposed method to rank the performance of various stocks by examining them under certain important criteria in financial trading is discussed. The stock market is one of the significant areas where unpredictability is at crest. The portfolio selection problem is multi-faceted in nature. So, a multiple-criteria decision making approach is used to solve this underlying problem. In the process of portfolio selection, there are broadly two stages: (a) some suitable shares are chosen; (b) the percentage of total investment for each share is obtained through different weighing schemes or through optimization techniques.

3.1 Structure of criteria and alternatives

Similar to all decision-making processes there are many fundamental factors which influence selection of a stock and makes our decision-making problem so complex such as P/E, P/BV, EPS, Beta, ROE etc. According to the decision makers view point, we have selected four factors as criteria. Real data for 20 stocks is collected from the http://www.moneycontrol.com and http://www.ratestar.in for Jan-2010 to Dec-2020. Since data is multi-dimensional, a method of "Exponential moving average (EMA)" is used to convert multi-dimensional data into a deterministic value.

Table 9. Evaluation criteria for stock selection

Evaluation Criteria	Definition of criteria
C_1: EPS	It shows how much of a company's profit after tax that each shareholder owns.
C_2: ROE	Return on Equity measures the return or profit earned per share by equity holder. Company having high ROE consider good for investment.
C_3: Revenue	The increasing rate of revenue indicates the increasing demand of company's product in the market.
C_4: P/E	It tells the stocks of the company are overvalued or not.

So, the three criteria EPS (C_1), ROE (C_2) and Revenue (C_3) are considered as beneficial-criteria by decision maker. P/E (C_4) is taken as non-beneficial or cost-criteria. We have selected the following 10 stocks out of 20 stocks that meet the selected criteria:

1. Pidilite Industries Ltd (st_1);
2. Tata Consultancy Services Ltd (st_2);
3. Reliance Industries Ltd (st_3);
4. Hindustan Unilever Ltd (st_4);
5. Asian Paints Ltd (st_5);
6. Bajaj Finance Ltd (st_6);
7. Infosys Ltd (st_7);
8. Aarti Industries Ltd (st_8);
9. Adani ports and Special Economic Zone Ltd (st_9);
10. Jubilant Foodworks Ltd (st_{10});

Table 10. EMA of actual numerical values of each criterion

Alternatives/Criteria	$C_1(EPS)$	$C_2(ROE)$	$C_3(Revenue)$	$C_4(P/E)$
st_1	18	25	6699	64
st_2	73	36	141259	25
st_3	55	10	511664	22
st_4	26	79	37695	71
st_5	22	18	18435	65
st_6	57	18	18342	43
st_7	34	24	86331	24
st_8	23	19	3800	30
st_9	17	18	11207	20
st_{10}	17	22	3377	84

3.2 Selection of stocks based on fuzzy MEREC-TOPSIS

Decision makers assigned the performance rating to alternatives on every criterion as per Table 10. The Table 11 demonstrates the judgement made by decision maker.

Table 11. Judgement matrix by decision maker

Alternatives/Criteria	C_1	C_2	C_3	C_4
st_1	EL	MH	VSL	ML
st_2	EH	VH	VSH	VSH
st_3	SH	EL	EH	VSH
st_4	SL	EH	VH	SL
st_5	VSL	SL	ML	ML
st_6	SH	SL	ML	VH
st_7	MH	MH	SH	VSH
st_8	VSL	SL	EL	SH
st_9	EL	SL	SL	EH
st_{10}	EL	ML	EL	EL

These results are converted to their respective TFNs using Table 1 to obtain the fuzzy decision-making matrix. By make use of Eq. 4, the normalized fuzzy decision matrix (Table 12) is then constructed.

Table 12. Normalized fuzzy decision matrix

Alternatives/Criteria	C_1	C_2	C_3	C_4
st_1	(0.00,0.00,0.00)	(0.43,0.50,0.57)	(0.00,0.13,0.14)	(0.29,0.38,0.43)
st_2	(1.00,1.00,1.00)	(0.57,0.63,0.71)	(0.86,0.88,1.00)	(0.86,0.88,1.00)
st_3	(0.71,0.75,0.86)	(0.00,0.00,0.00)	(1.00,1.00,1.00)	(0.86,0.88,1.00)
st_4	(0.14,0.25,0.29)	(1.00,1.00,1.00)	(0.57,0.63,0.71)	(0.14,0.25,0.29)
st_5	(0.00,0.13,0.14)	(0.14,0.25,0.29)	(0.29,0.38,0.43)	(0.29,0.38,0.43)
st_6	(0.71,0.75,0.86)	(0.14,0.25,0.29)	(0.29,0.38,0.43)	(0.57,0.63,0.71)
st_7	(0.43,0.50,0.57)	(0.43,0.50,0.57)	(0.71,0.75,0.86)	(0.86,0.88,1.00)
st_8	(0.00,0.13,0.14)	(0.14,0.25,0.29)	(0.00,0.00,0.00)	(0.71,0.75,0.86)
st_9	(0.00,0.00,0.00)	(0.14,0.25,0.29)	(0.14,0.25,0.29)	(1.00,1.00,1.00)
st_{10}	(0.00,0.00,0.00)	(0.29,0.38,0.43)	(0.00,0.00,0.00)	(0.00,0.00,0.00)

The weights of the criteria are calculated by using Eq. 5,6,7 and 8. In order to get the weighted normalized decision matrix the Eq.9 is used.

$\widetilde{w}_1 = (0.188, 0.228, 0.272)$; $\widetilde{w}_2 = (0.164, 0.201, 0.227)$; $\widetilde{w}_3 = (0.209, 0.254, 0.311)$;
$\widetilde{w}_4 = (0.262, 0.317, 0.405)$;

Table 13. The weighted normalized fuzzy decision matrix

Alternatives/Criteria	C_1	C_2	C_3	C_4
st_1	(0.00,0.00,0.00)	(0.07,0.10,0.13)	(0.00,0.03,0.04)	(0.07,0.12,0.17)
st_2	(0.19,0.23,0.27)	(0.09,0.13,0.16)	(0.18,0.22,0.31)	(0.22,0.28,0.40)
st_3	(0.13,0.17,0.23)	(0.00,0.00,0.00)	(0.21,0.25,0.31)	(0.22,0.28,0.40)
st_4	(0.03,0.06,0.08)	(0.16,0.20,0.23	(0.12,0.16,0.22)	(0.04,0.08,0.12)
st_5	(0.00,0.03,0.04)	(0.02,0.05,0.06)	(0.06,0.10,0.13)	(0.07,0.12,0.17)
st_6	(0.13,0.17,0.23)	(0.02,0.05,0.06)	(0.06,0.10,0.13)	(0.15,0.20,0.29)
st_7	(0.08,0.11,0.16)	(0.07,0.10,0.13)	(0.15,0.19,0.27)	(0.22,0.28,0.40)
st_8	(0.00,0.03,0.04)	(0.02,0.05,0.06)	(0.00,0.00,0.00)	(0.19,0.24,0.35)
st_9	(0.00,0.00,0.00)	(0.02,0.05,0.06)	(0.03,0.06,0.09)	(0.26,0.32,0.40)
st_{10}	(0.00,0.00,0.00)	(0.05,0.08,0.10)	(0.00,0.00,0.00)	(0.00,0.00,0.00)

Then according to Eq. 10, FPIS and FNIS are calculated as

$A^+ = \{(0.188, 0.288, 0.272), (0.164, 0.201, 0.227), (0.209, 0.257, 0.311)\}$

$A^- = \{(0,0,0), (0,0,0), (0,0,0)\}$

By make use of Eq. 11 and 12, we obtain s_i^+ and s_i^- and CC_i. The calculations are shown in Table 13.

Table 14. Calculation of s_i^+, s_i^- and CC_i

Alternatives	s_i^+	s_i^-	CC_i	Rank
st_1	0.694	0.291	0.295	9
st_2	0.140	0.862	0.855	1
st_3	0.360	0.773	0.682	3
st_4	0.555	0.489	0.468	6
st_5	0.637	0.300	0.320	8
st_6	0.442	0.533	0.547	4
st_7	0.281	0.704	0.715	2
st_8	0.647	0.471	0.421	7
st_9	0.590	0.594	0.502	5
st_{10}	0.863	0.132	0.132	10

The ranking order of alternatives based on Table 14 is:

$$st_2 > st_7 > st_3 > st_6 > st_9 > st_4 > st_8 > st_5 > st_1 > st_{10}$$

Hence, alternative st_2 is the most preferable one.

3.3 Portfolio analysis

The main objective of stock portfolio selection is to distribute capital to several selected stocks to get the most profitable returns for investors. In this sub-section, we conduct a portfolio analysis based on the results obtained in previous section. Ranked stocks i.e., $st_2, st_7, st_3, st_6, st_9, st_4, st_8, st_5, st_1, st_{10}$ are taken as the assets of the portfolio. We assigned the weights according to the rank of assets. Investor invests more on high ranked asset than low ranked. Hence, the assigned weight or the share of total investment on the stocks $st_2, st_7, st_3, st_6, st_9, st_4, st_8, st_5, st_1, st_{10}$ taken as 0.15, 0.14, 0.13, 0.12, 0.11, 0.1, 0.09, 0.07, 0.05, 0.04 respectively.

To calculate the return of the portfolio, we have taken historical data (Jan-2014 to June-2021) of each asset from www.yahoofinance.com. The expected return of the portfolio is mentioned in Table 15.

Table 15. Portfolio return based on the proposed model

Alternatives	st_2	st_7	st_3	st_6	st_9	st_4	st_8	st_5	st_1	st_{10}
Avg. monthly ret.	0.015	0.017	0.022	0.050	0.024	0.019	0.048	0.022	0.024	0.025
Avg. annual ret.	0.184	0.205	0.269	0.602	0.290	0.233	0.579	0.269	0.293	0.308
Expected return	0.033	0.035	0.043	0.090	0.038	0.025	0.057	0.018	0.014	0.012
Portfolio return	0.3168									

In order to verify the reliability of the proposed model, we have forecast future data to test the performance of the portfolio in the future, for which we have gathered data of each stock from 01-January-2018 to 18-June-2021 from www.yahoofinance.com. We have used deep recurrent neural network for forecasting future prices of 30 days. The daily closing price of stocks is used to train the network. The neural network comprises of 5 layers having 1 input layer with 50 nodes, 3 hidden layers having 25 nodes each and 1 output layer having 1 node that predicts the future price of the next day. The neural network has been trained for 100 epochs and predicted future prices of each stock. The forecasted data is shown in the Table 16. The results of the future analysis validate that portfolio is very reliable and ensures the applicability and universality of the proposed model by assuring better portfolio returns in future.

The results of the developed method are also compared with the results of the previously existing methods. Table 17 promotes the stability of the ranking system proposed in this paper.

Table 16. Closing price of next 30 days forecast by deep recurrent neural network (LSTM)

st_1	st_2	st_3	st_4	st_5	st_6	st_7	st_8	st_9	st_{10}
2103.37	3300.94	2219.17	2476.85	3041.16	6044.52	1493.70	1843.55	704.16	3243.98
2106.96	3315.59	2209.18	2447.72	3042.37	6014.91	1502.92	1835.30	702.66	3255.20
2102.26	3326.07	2200.41	2445.85	3043.96	5984.69	1510.22	1830.88	705.81	3284.94
2097.53	3332.74	2193.96	2449.47	3045.06	5975.06	1517.26	1824.77	710.44	3297.23
2091.95	3338.28	2189.05	2441.98	3045.67	5971.82	1519.90	1825.88	713.94	3303.98
2084.82	3343.75	2184.63	2434.43	3046.10	5947.44	1520.53	1848.46	717.04	3294.78
2083.31	3348.87	2180.38	2431.80	3046.49	5929.41	1527.23	1844.34	720.11	3309.12
2079.25	3353.34	2176.39	2429.13	3046.87	5915.28	1532.43	1842.87	723.05	3324.34
2075.13	3357.21	2172.75	2425.05	3047.19	5906.92	1536.45	1843.71	725.78	3330.96
2071.05	3360.62	2169.47	2421.61	3047.47	5897.08	1539.11	1848.66	728.34	3334.92
2067.36	3363.64	2166.50	2419.03	3047.71	5882.54	1542.25	1855.43	730.75	3338.32
2064.78	3366.32	2163.80	2416.46	3047.91	5871.46	1546.95	1854.23	733.00	3349.15
2061.49	3368.67	2161.32	2413.89	3048.09	5862.51	1550.63	1854.95	735.09	3357.82
2058.32	3370.74	2159.05	2411.61	3048.24	5854.74	1553.68	1857.18	737.05	3363.04
2055.36	3372.56	2156.98	2409.56	3048.37	5846.00	1556.50	1860.33	738.86	3367.99

Table 16. Closing price of next 30 days forecast by deep recurrent neural network (LSTM) (continued from the previous page)

st_1	st_2	st_3	st_4	st_5	st_6	st_7	st_8	st_9	st_{10}
2052.69	3374.16	2155.08	2407.63	3048.48	5836.81	1559.68	1862.66	740.55	3374.00
2050.22	3375.57	2153.35	2405.81	3048.58	5829.30	1563.04	1862.94	742.11	3381.67
2047.63	3376.81	2151.75	2404.15	3048.66	5822.64	1565.86	1864.23	743.56	3387.74
2045.19	3377.89	2150.29	2402.61	3048.74	5816.19	1568.44	1866.04	744.90	3392.71
2042.89	3378.85	2148.95	2401.17	3048.80	5809.58	1571.01	1867.78	746.13	3397.93
2040.73	3379.68	2147.72	2399.83	3048.85	5803.32	1573.63	1868.94	747.27	3403.62
2038.64	3380.42	2146.59	2398.59	3048.90	5797.84	1576.16	1869.68	748.32	3409.37
2036.59	3381.06	2145.55	2397.43	3048.93	5792.69	1578.42	1870.83	749.28	3414.31
2034.64	3381.63	2144.60	2396.35	3048.97	5787.68	1580.58	1872.04	750.17	3418.96
2032.79	3382.13	2143.72	2395.35	3049.00	5782.81	1582.71	1873.04	750.99	3423.75
2031.29	3382.56	2142.92	2394.41	3049.02	5778.29	1584.79	1873.86	751.73	3428.57
2029.29	3382.94	2142.18	2393.54	3049.04	5774.14	1586.74	1874.59	752.42	3433.17
2027.63	3383.28	2141.49	2392.72	3049.06	5770.18	1588.55	1875.44	753.05	3437.41
2026.04	3383.57	2140.87	2391.96	3049.08	5766.37	1590.30	1876.23	753.63	3441.55
2024.52	3383.83	2140.29	2391.25	3049.09	5762.75	1592.00	1876.91	754.16	3445.68

Table 17. Comparison of proposed model with earlier studies

	Model (Thakur et al.,2018)	Model (Singh, 2020)	Proposed model
Year	2016.	2020.	2021.
Ex. Return	0.1301	0.050	0.3168

4. Conclusion

Because of the ambiguity and inference of decision-making data, applying fuzzy sets to MCDM methods can lead to more reliable decision-making results. In this paper, the MEREC method is developed in ambiguous environment. To perform this task, linguistic terms have been used which can be converted into their respective TFNs. The performance measure function plays a vital role in the determination of weight by MEREC. A parabolic measure has been used in the expansion of MEREC in ambiguous environment as it is fully capable to work according to the properties of TFNs. In the field of decision-making methods, rational and meaningful unification of some rules and strategies adds special benefits in knowledge. MEREC is much capable of obtaining values of relative weights of multiple conflicting criteria using removal effects of criteria on alternative's performance. The TOPSIS method is able to measure the relative performance of each alternative in simple mathematical terms because of its good computational efficiency. So, in the second part, a new hybrid fuzzy MEREC-TOPSIS method is proposed to take advantage of both the methods simultaneously. The unification of MEREC with TOPSIS in fuzzy circumstances makes complex decision-making process much more efficient, accurate and flexible.

A new hybrid ranking model "fuzzy MEREC-TOPSIS" has major contribution and novelties as follows:

- The fuzzy MEREC is fully capable of considering the ambiguity of the decision maker in the decision-making process. The combination of the three (simple normalization technique, parabolic measure and Euclidean distances) makes the fuzzy MEREC simple and efficient.
- In the second part of presented study, a novel hybrid ranking method is introduced that completes the aim of evaluating and selecting the most preferrable alternative under fuzzy environment.

- The proposed method is less computational as the decision makers have to provide the performance rating only once based on the initial data to calculate the weighting of the criteria and the ranking of the alternatives.
- Using the proposed method, a real case study of stock portfolio selection is discussed. The case study addresses the significance and advantages of the method to accomplish the objective of the paper.

Integrating MEREC with other objective and subjective weighting methods can be the focus in future research. Researchers could use the generalizations of fuzzy set such as pythagorean, intuitionistic, type-2, hesitant etc. Although the method is proposed and tested for stock selection, it can also be applied in different real-world decision-making problems such as supplier selection, management and engineering applications. Here, the model is implemented for NSE, it can be applied for portfolio building in any exchange.

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