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A heuristic approach for fuzzy fixed charge transportation problem

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Abstract

The fixed charge transportation problem is a nonlinear programming problem in which a fixed charge is incurred if a distribution variable assumes a nonzero value. Its structure is almost identical to that of a linear programming problem, and indeed it can be written as a mixed-integer linear program. In this paper, we consider the fixed charge transportation problem under uncertainty, particularly when the direct and fixed costs are the generalized trapezoidal fuzzy numbers. The first step it transforms into the classical fuzzy transportation problem. The next we obtain the best approximation fuzzy on the optimal value of the fuzzy fixed-charge transportation problem. This method obtains a lower and upper bound both on the fuzzy optimal value of the fuzzy fixed-charge transportation problem which can be easily obtained by using the approximation solution. Conclusion: we propose a new method as the best approximation method, with representation both of the transportation cost and the fixed cost, to find a fuzzy approximation solution close to the optimal solution for fuzzy fixed charge transportation problem.

Keywords: fixed-charge transportation, generalized trapezoidal fuzzy numbers, fuzzy transportation problem

1. Introduction

Transportation models have wide applications in the real world situations (Hasani and Moghimi, 2014). A special version of the transportation problem (TP) is the fixed-charge transportation problem (FCTP). In the FCTP, each route is associated with a fixed cost and a transportation cost per unit shipped. Since problems with fixed charge are usually NP-hard problems (Adlakha and Kowalski, 2010), the computational time to obtain exact solutions increases in the distinguished Class P of problems and very quickly become extremely long as the size of the problem increase (Adlakha and Kowalski, 2010). Thus, any method which provides a good solution should be considered useful (Kiyoumarsi and Asgharian, 2014).

According to the available literature, a wide range of different strategies are used in order to find an optimal solution for FCTPs. Generally, the solving methods of the FCTP can be classified as: exact, heuristic and meta-heuristic methods.

Many researchers attempted to solve the small size FCTP using heuristic methods. Although heuristic methods are usually computationally efficient, the major disadvantage of heuristic methods is the possibility of terminating at a local optimum that is far distant from the global optimum. And, the meta-heuristic methods were proposed to solve such hard optimization problems (Hajiaghaei-Keshteli et al., 2010; Molla-Alizadeh-Zavardehi et al., 2011; Shirvani et al., 2014; Khoshfetrat et al., 2014; Ebrahimi et al., 2014).

All of the aforementioned literatures briefly introduce the FCTP concept in an effort to familiarize the reader with the underlying theory and then present the approaches with precise data to solve the FCTP. In fact, for each possible transportation pattern in the real world, some or all the parameters are not only well-defined, precise data, but also vague or fuzzy data (Marashi Aliabadi and Gholizadeh Zahmatkesh, 2014). Zadeh (1965) presented the fuzzy set theory for the first time to handle the unclarity of human's decision making (Rahimi Ghazikalayeh et al., 2014; Tabadar and Tabadar, 2014; Moradi Majd et al., 2014). The role of fuzzy sets in decision processes is best described in the original statements of Bellman and Zadeh (1970). Thus, decision processes are better described and solved by using fuzzy set theory, rather than precise approaches (Lai and Hwang, 1992). To this end, the application of the fuzzy set theory to the linear programming and multi-criteria decision making problems was proposed by Zimmermann (1978). Chanas et al. (1984) presented a fuzzy linear programming model to solve TP with fuzzy supply and demand values. Chanas and Kuchta (1996) developed an algorithm to obtain the optimal solution based on type of TPs with fuzzy coefficients. Kaur and Kumar (2012) tried to propose a new approach to solve a special type of fuzzy TPs by representing the transportation costs as generalized trapezoidal fuzzy numbers (GTFNs). Kaur and Kumar (2012) proposed a new method to solve the fuzzy TPs, where transportation cost, availability and demand of the products are represented by the GTFNs.

As far as we know, with regard to solve the fuzzy fixed-charge transportation problem (FFCTP), no research has been done. Therefore, any method which provides a good solution for it will be distinguished. In order to, the present paper, first, tries to convert the FFCTP into the fuzzy transportation problem (FTP) by using the development of Balinski's formula. This becomes a linear version of the FFCTP for the next stage, and then, tries to obtain a fuzzy initial basic feasible solution and optimal solution both of the linear version of the FFCTP by using one of the well-known methods, such as generalized north-west corner method, generalized fuzzy least-cost method, generalized fuzzy Vogel's approximation method and fuzzy modified distribution method (Kaur and Kumar, 2012).

Most of the literatures on the FTP topic are only concerned with the normal fuzzy numbers instead of the generalized fuzzy numbers. They, first, try to convert the generalized fuzzy numbers into the normal fuzzy numbers by using the normalization process and then try to solve the real life problems by considering them. There is a serious disadvantage of the normalization process (Kaufmann and Gupta, 1985). But in many real-world applications, it is not possible to restrict the membership function to the normal form, and we should avoid it. To this end, a method which is called the best approximation method is proposed to find an approximation solution close to the optimal solution for the FFCTP when the transportation cost and fixed cost are the GTFNs. The proposed method obtains a lower and upper bounds both on the fuzzy optimal value of the FFCTP which can be easily obtained by using the approximation solution. This is an important advantage of the proposed method.

The rest of the paper is organized as follows: in Section 1, some basic definitions and arithmetic operations between two the GTFNs are reviewed. Then, formulation of the fixed-charge transportation problem is recalled. Later, the FFCTP is presented. In the next section, we proposed the best approximation method to the FFCTP. To explain the method, a numerical example is solved in section 3. Finally, conclusions are pointed out in the last section.

2. Preliminaries

In this section, we briefly review some fundamental definitions and basic notation of the fuzzy set theory in which will be used in this paper.

Definition 1. (Kaufmann and Gupta, 1988): If X is a collection of objects denoted generically by x, then a fuzzy set in X is a set of ordered pairs, $\tilde{A}=\{(x,\tilde{A}(x))|x\in X\}$, where $\tilde{A}(x)$ is called the membership function which associates with each $x \in X$ a number in [0, 1] indicating to what degree x is a number.

Definition 2. (Kaufmann and Gupta, 1988): A fuzzy set $\tilde{A}$ on $\mathbb{R}$ is a fuzzy number if the following conditions hold:

(a) Its membership function is piecewise continuous function.

(b) There exist three intervals [a, b], [b, c] and [c, d] such that $\tilde{A}$ is strictly increasing on [a, b], equal to 1 on [b, c], strictly decreasing on [c, d] and equal to 0 elsewhere.

Definition 3. (Kaufmann and Gupta, 1988): A fuzzy number $\tilde{A}$ is said to be a trapezoidal fuzzy number (TFN) if its membership function is given by

$$\tilde{A}(x)=\begin{cases} \dfrac{x-a}{b-a}, & a\le x<b \\ 1, & b\le x\le c \\ \dfrac{x-d}{c-d}, & c<x\le d \\ 0, & otherwise \end{cases}$$

Definition 4. (Chen and Chen, 2007): A fuzzy set $\tilde{A}$, defined R, is said to be generalized fuzzy number if the following conditions hold:

(a) Its membership function is piecewise continuous function.

(b) There exist two intervals [a, b] and [c, d] such that $\tilde{A}$ is strictly increasing on [a, b] and strictly decreasing on [c, d].

(c) $\tilde{A}(x) = w$, For all $x \in [a,b]$ where $0 \le w < 1$.

Definition 5. (Chen and Chen, 2007): A fuzzy number $\tilde{A}=(a,b,c,d;w)$ is said to be a generalized trapezoidal fuzzy number (GTFN) if its membership function is given by

$$\tilde{A}(x)=\begin{cases} w\dfrac{x-a}{b-a}, & a\le x<b \\ w, & b\le x\le c \\ w\dfrac{x-d}{c-d}, & c<x\le d \\ 0, & otherwise \end{cases}$$

If w = 1, then the GTFN $\tilde{A}=(a,b,c,d;w)$ is called a TFN and denoted as $\tilde{A} = (a, b, c, d)$.

In this subsection, we reviewed arithmetic operations on GTFNs (Chen and Chen, 2007).

Let $\tilde{A}=(a_1,b_1,c_1,d_1;w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two GTFNs. Define,

$\tilde{A}\oplus\tilde{B}=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2;w)$

where $w=\min\{w_1,w_2\}$,

$\tilde{A}\ominus\tilde{B}=(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2;w)$,

where $w=\min\{w_1,w_2\}$,

$$\theta\tilde{A}=\begin{cases}(\theta a_1,\theta b_1,\theta c_1,\theta d_1;w_1) & \theta>0,\\(\theta d_1,\theta c_1,\theta b_1,\theta a_1;w_1) & \theta<0,\end{cases}$$

2.1. Ranking function

A ranking function is suited to compare the fuzzy numbers (Saeednamaghi and Zare, 2014). A ranking function is defined as, $R:F(\mathbb{R})\to\mathbb{R}$, where $\mathrm{F(R)}$ is a set of fuzzy numbers, that is, a mapping which maps each fuzzy number into the real line. Now, suppose that $\tilde{\mathrm{A}}$ and $\tilde{B}$ be two GTFNs. Therefore,

1. $R(\tilde{\mathrm{A}})>R(\tilde{\mathrm{B}})$ if $\tilde{\mathrm{A}}\succ\tilde{\mathrm{B}}$ i.e., minimum $\{\tilde{\mathrm{A}},\tilde{\mathrm{B}}\}=\tilde{\mathrm{B}}$,
2. $\mathrm{R}(\tilde{\mathrm{A}})<\mathrm{R}(\tilde{\mathrm{B}})$ if $\tilde{A}<\tilde{B}$ i.e., minimum$\{\tilde{\mathrm{A}},\tilde{\mathrm{B}}\}=\tilde{\mathrm{A}}$,
3. $\mathrm{R}(\tilde{\mathrm{A}})=\mathrm{R}(\tilde{\mathrm{B}})$ if $\tilde{\mathrm{A}}\approx\tilde{\mathrm{B}}$ i.e., minimum$\{\tilde{\mathrm{A}},\tilde{\mathrm{B}}\}\approx\tilde{\mathrm{A}}\approx\tilde{\mathrm{B}}$.

Remark 1. (Chen and Chen, 2007). Let $\tilde{\mathrm{A}}=(\mathrm{a}_1,\mathrm{b}_1,\mathrm{c}_1,\mathrm{d}_1;\mathrm{w}_1)$ be any GTFN, Then

$$R(\tilde{\mathrm{A}})=w_1(\frac{a_1+b_1+c_1+d_1}{4})$$

Now, let $\tilde{\mathrm{A}}=(\mathrm{a}_1,\mathrm{b}_1,\mathrm{c}_1,\mathrm{d}_1;\mathrm{w}_1)$ and $\tilde{\mathrm{B}}=(\mathrm{a}_2,\mathrm{b}_2,\mathrm{c}_2,\mathrm{d}_2;\mathrm{w}_2)$ be two GTFNs, then to compare $\tilde{\mathrm{A}}$ and$\tilde{\mathrm{B}}$, we use the following steps (Kaur and Kumar, 2012):

1. Find w = minimum (w_1 , w_2).
2. Find $R(\tilde{\mathrm{A}})=w(\frac{a_1+b_1+c_1+d_1}{4})$, and $R(\tilde{\mathrm{B}})=w(\frac{a_2+b_2+c_2+d_2}{4})$.
3. a) If $R(\tilde{\mathrm{A}})>R(\tilde{\mathrm{B}})$ then $\tilde{\mathrm{A}}\succ\tilde{\mathrm{B}}$,
   b) If $\mathrm{R}(\tilde{\mathrm{A}})<\mathrm{R}(\tilde{\mathrm{B}})$ then $\tilde{\mathrm{A}}\prec\tilde{\mathrm{B}}$,
   c) If $\mathrm{R}(\tilde{\mathrm{A}})=\mathrm{R}(\tilde{\mathrm{B}})$ then $\tilde{\mathrm{A}}\approx\tilde{\mathrm{B}}$.

3. Fixed charge transportation problem

Consider a TP with m sources and n destinations. Each of the source i=1,2,…,m has $\mathrm{S_i}$ units of supply, and each the destination j=1,2,…,n has a demand of $\mathrm{D_j}$units and also, each of the m source can ship to any of the n destinations at a shipping cost per unit c_{ij} plus a fixed cost $\mathrm{f_{ij}}$ assumed for opening this route *(i,j)*. Let x_{ij} denote the number of units to be shipped from the source *i* to the destination *j*. We need to determine which routes are to be opened and the size of the shipment on those routes, so that the total cost of meeting demand, given the supply constraints, is minimized. Then, the FCTP is the following mixed integer programming problem (Balinski, 1961):

$$\begin{aligned}&Min\sum_{i=1}^{m}\sum_{j=1}^{n}(\mathrm{c}_{ij}\mathrm{x}_{ij}+f_{ij}\,\mathrm{y}_{ij})\\&s.t.\\&\sum_{j=1}^{n}x_{ij}=S_i,\ i=1,2,\dots,m\\&\sum_{i=1}^{m}x_{ij}=D_j,\ j=1,2,\dots,n\\&x_{ij}\ge0,\ i=1,2,\dots,m\ ,\ j=1,2,\dots,n\\&y_{ij}=\begin{cases}1, & x_{ij}>0,\\0, & otherwise\end{cases}\end{aligned}\tag{1}$$

where c_{ij} and f_{ij} are the real numbers.

Without losing generality, we assume that the TP is balanced. Let TP be unbalanced, then by introducing a dummy source or a dummy destination it can be converted to a balanced TP. Despite of its similarity to the conventional TP, the FCTP is significantly harder to solve because of the discontinuity in the objective function introduced by the fixed costs.

4. Fuzzy fixed-charge transportation problem

Now, we assume that the transportation cost and the fixed cost to open a route *(i,j)* denote by c_{ij} and f_{ij}, respectively, which are not deterministic numbers, but they are the GTFNs, so, total transportation costs become fuzzy as well. The fuzzy fixed-charge transportation problem (FFCTP) is the following mathematical form:

$$\begin{aligned}
& Min \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}x_{ij} \oplus \tilde{f}_{ij}\, y_{ij}) \\
& s.t. \\
& \sum_{j=1}^{n} x_{ij} = S_i,\ i=1,2,\ldots,m, \\
& \sum_{i=1}^{m} x_{ij} = D_j,\ j=1,2,\ldots,n, \\
& x_{ij} \geq 0,\ i=1,2,\ldots,m,\ j=1,2,\ldots,n, \\
& y_{ij} = \begin{cases} 1, & x_{ij} > 0, \\ 0, & otherwise \end{cases},\ i=1,2,\ldots,m\ ,\ j=1,2,\ldots,n.
\end{aligned} \tag{2}$$

where, $\tilde{c}_{ij}$ and $\tilde{f}_{ij}$ are the GTFNs.

Balinski (1961) proposed an approximation solution with heuristic method for the FCTP. This paper tries to develop the Balinski's heuristic method for the FFCTP. To do so, first, suppose that both of the transportation cost and the fixed cost are GTFNs as $\tilde{c}_{ij}$, respectively, then the Balinski matrix is obtained by formulating a linear version of the FFCTP by relaxing the integer restriction on y_{ij} in the objective function of model (2) as follows:

$$y_{ij} = \frac{x_{ij}}{M_{ij}}$$

where $M_{ij} = \min\{S_i, D_j\}$.

So, the linear version of the FFCTP can be represented as follows:

$$\begin{aligned}
& Min \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \oplus \frac{\tilde{f}_{ij}}{M_{ij}})x_{ij} \\
& s.t. \\
& \sum_{j=1}^{n} x_{ij} = S_i,\ i=1,2,\ldots,m \\
& \sum_{i=1}^{m} x_{ij} = D_j,\ j=1,2,\ldots,n \\
& x_{ij} \geq 0,\ i=1,2,\ldots,m\ ,\ j=1,2,\ldots,n
\end{aligned} \tag{3}$$

We call this "the Approximation Fuzzy Transportation Problem (AFTP)" in which the unit transportation cost is recalculated according to:

$$C_{ij} = \tilde{c}_{ij} \oplus \frac{\tilde{f}_{ij}}{M_{ij}}$$

The AFTP is the classical FTP with the fuzzy transportation costs.

Assume that, $\{x^*_{ij}\}$ is the optimal solution of the AFTP. It can be easily modified into a feasible solution $\{x'_{ij}, y'_{ij}\}$ of (2) as follows:

$$x'_{ij} = y'_{ij} = 0 \ \text{ if } \ x^*_{ij} = 0,, \text{ and } x'_{ij} = x^*_{ij} \text{ and } \ y'_{ij} = 1 \ \text{ if } \ x^*_{ij} > 0..$$

Theorem 1. The optimal value of the AFTP provides a lower bound to the optimal objective value of problem (2).

Proof. Let $\{x^*_{ij}\}$ be an arbitrary optimal solution of the AFTP, and $\{\bar{x}_{ij}, \bar{y}_{ij}\}$ be an optimal solution for (2), where $\bar{y}_{ij} = 1$ if $\bar{x}_{ij} > 0$. Since $\{\bar{x}_{ij}\}$ is a feasible solution of the AFTP, therefore,

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \oplus \frac{\tilde{f}_{ij}}{M_{ij}})x^*_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \oplus \frac{\tilde{f}_{ij}}{M_{ij}})\bar{x}_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}\,\bar{x}_{ij} \oplus \frac{\tilde{f}_{ij}}{M_{ij}}\bar{x}_{ij})$$

$$\leq \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}\,\bar{x}_{ij} \oplus \frac{\tilde{f}_{ij}}{\bar{x}_{ij}}\bar{x}_{ij}) \qquad Since \qquad \bar{x}_{ij} \leq M_{ij}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}\,\bar{x}_{ij} \oplus \bar{f}_{ij}\bar{y}_{ij}) \qquad Since \qquad \bar{y}_{ij} = 1 = \frac{\bar{x}_{ij}}{\bar{x}_{ij}}.$$

Theorem 2. Suppose that $\{\bar{x}'_{ij}, \bar{y}'_{ij}\}$ is an arbitrary feasible solution of the FFCTP. Then, the objective value of $\{\bar{x}'_{ij}, \bar{y}'_{ij}\}$ of (2) provides an upper bound to the optimal value of (2).

Proof. Its Proof is straightforward.

Corollary 1. According to the above theorems, the optimal value of the FFCTP $(\tilde{Z}^*_{FFCTP})$ is between the optimal value of the AFTP $(\tilde{Z}^*_{AFTP})$ and the objective value of an arbitrary feasible solution of the FFCTP $(\tilde{Z}_{FFCTP})$. That is,

$$Z^*_L = \tilde{Z}^*_{AFTP} \leq \tilde{Z}^*_{FFCTP} \leq \tilde{Z}_{FFCTP} = \tilde{Z}^*_U.$$

Corollary 2. Let $\{x'_{ij}, y'_{ij}\}$ be a feasible solution of (2), and using this solution $\tilde{Z}^*_L = \tilde{Z}^*_U$. Then, $\{x'_{ij}, y'_{ij}\}$ is an optimal solution of (2) and $\tilde{Z}^*_L = \tilde{Z}^*_{FFCTP} = \tilde{Z}^*_U$.

5. Results

This section proposes a method as the best approximation method, to find an approximation solution to the optimal solution of the FFCTP. Its steps are as follow:

Step 1. Convert the given the FFCTP into the FTP as the AFTP by using the following formula:

$$\tilde{C}_{ij} = \frac{\tilde{f}_{ij}}{M_{ij}} \oplus \tilde{c}_{ij}$$

where $M_{ij} = \min\{S_i, D_j\}$.

Step 2. Apply one of the well-known methods, as such generalized north-west corner method, the generalized fuzzy least-cost method, or the generalized fuzzy Vogel's approximation method (Kaur and Kumar, 2012) to obtain an initial basic feasible solution of the AFTP.

Step 3. Apply fuzzy modified distribution method (Kaur and Kumar, 2012) to obtain a fuzzy optimal solution of the AFTP.

Step 4. Provide a lower bound ($\tilde{Z}_L^*$) on the optimal value of the FFCTP ($\tilde{Z}_{FFCTP}^*$) according to the theorem 1, by calculating the optimal value of the AFTP.

Step 5. Provide an upper bound ($\tilde{Z}_U^*$) on the optimal value of the FFCTP ($\tilde{Z}_{FFCTP}^*$) according to theorem 2, by calculating the objective value of an arbitrary feasible solution of the FFCTP.

6. Discussion

Suppose that a company has three factories in three different cities of 1, 2 and 3. The goods of these factories are assembled and sent to the major markets in the three other cities. The demand ($S_i, i=1,2,3$), supply ($D_j, j=1,2,3$)) for the cities and the transportation cost associated with each route *(i,j)* are given by the Table 1. Let's also assume that there is a fixed cost in this transportation problem. Namely, the cost of sending no units along route *(i,j)* is zero; but any positive shipment incurs a fixed cost plus a cost proportional to the number of unites transported. Notice that both quantities of the transportation cost ($\tilde{c}_{ij}, i,j=1,2,3$) and the fixed cost ($\tilde{f}_{ij}$, i, j=1,2,3) are fuzzy numbers in this example as shown by the Table 1.

Table 1. The fuzzy transportation costs and the fuzzy fixed costs ($\tilde{c}_{ij}, \tilde{f}_{ij}$) for the numerical example

	$D_1=10$	$D_2=30$	$D_3=10$
$S_1=15$	$S_2=20$	((1,2,5,9;0.4), (7,9,13,20;0.3))	((2,5,8,18;0.5), (3,8,18,28;0.1))
$S_2=20$	$S_3=15$	((3,5,8,12;0.2), (8,13,17,20;0.4))	((7,9,13,28;0.4),(6,18,25,40;0.2))
$S_3=15$	((11,12,20,27;0.5), (0,3,8,10;0.2))	((0,5,10,15;0.8), (5,7,18,23;0.5))	((4,5,8,11;0.6), (7,17,20,28;0.3))

The above problem is balanced, because, $\sum_{i=1}^{3} S_i = \sum_{j=1}^{3} D_j = 50$. A lower and upper bounds both for the fuzzy optimal value of the FFCTP in the given example by using the approximation method, proposed in section 5, can be obtained as follows.

Step 1. The transportation Table with fuzzy quantities for the cost of the problem using $\tilde{C}_{ij} = \frac{\tilde{f}_{ij}}{M_{ij}} \oplus \tilde{c}_{ij}$ is shown in Table 2.

Table 2. The transportation table of numerical example

	$D_1=10$	$D_2=30$	$D_3=10$
$S_1=15$	(1.3, 4.5, 9.8, 20; 0.3)	(1.47, 2.6, 5.87, 10.33; 0.3)	(2.3, 5.8, 9.8, 20.8; 0.1)
$S_2=20$	(8.2, 9.5, 12.9, 27.3; 0.5)	(3.4, 5.65, 8.85, 13; 0.2)	(7.6, 10.8, 15.5, 32; 0.2)
$S_3=15$	(11, 12.3, 20.8, 28; 0.2)	(0.33, 5.47, 11.2, 16.53; 0.5)	(4.7, 6.7, 10, 13.8; 0.3)

Step 2. The initial solution of the AFTP with the generalized north-west corner method is shown in Table 3.

Table 3. The initial solution of numerical example

	$D_1=10$	$D_2=30$	$D_3=10$
$S_1=15$	10	5	
$S_2=20$		20	
$S_3=15$		5	10

Step 3. We use the generalized fuzzy modified distribution method, to find the fuzzy optimal value of the AFTP. The value of the fuzzy dual variable $\tilde{u}_i s$ and $\tilde{v}_j s$ are computed with $\tilde{u}_i \oplus \tilde{v}_j = \tilde{c}_{ij}$, for any basic cell, we have:

$\tilde{u}_1 \oplus \tilde{v}_1 = (1,3,4.5,9.8;0.3)$, $\tilde{u}_1 \oplus \tilde{v}_2 = (1.47,2.6,5.87,10.33;0.3)$,

$\tilde{u}_2 \oplus \tilde{v}_2 = (3.4,5.65,8.85,13;0.2)$, $\tilde{u}_3 \oplus \tilde{v}_2 = (0.33,5.47,11.2,16.53;0.5)$,

$\tilde{u}_3 \oplus \tilde{v}_3 = (4.7,6.7,10,13.8;0.3)$.

To solving the above fuzzy system of equation, we set $\tilde{u}_1 = (0,0,0,0;1)$, therefore,

$\tilde{u}_1 = (1.3,4.5,9.8,20;0.3)$, $\tilde{v}_2 = (1.47,2.6,5.87,10.33;0.3)$,

$\tilde{u}_2 = (-6.93,-0.22,6.25,11.53;0.2)$, $\tilde{u}_3 = (-10,-0.4,8.6,15.06;0.3)$,

$\tilde{v}_3 = (-10.36,1.4,7.1,14.7;0.3)$.

To compute $\tilde{d}_{ij} = \tilde{C}_{ij} \ominus (\tilde{u}_i \oplus \tilde{v}_j)$, for each non-basic cell, we have:

$\tilde{d}_{21} = (-23.33,-6.55,8.62,32.93;0.2)$, $\tilde{d}_{31} = (-24.06,-6.1,16.7,36.7;0.2)$,

$\tilde{d}_{13} = (-12.7,-1.3,8.4,31.16;0.1)$, $\tilde{d}_{23} = (-18.63,-2.55,13.7,49.29;0.2)$,

Since $R(\tilde{d}_{ij}) \geq 0$, $\forall i, j$, therefore, this solution is optimal.

Step 4. A lower bound ($\tilde{Z}_L^*$) on the optimal value of the FFCTP ($\tilde{Z}_{FFCTP}^*$) by calculating the optimal value of the AFTP is as follows:

$\tilde{Z}_L^* = 10\tilde{C}_{11} + 5\tilde{C}_{12} + 20\tilde{C}_{22} + 5\tilde{C}_{32} + 10\tilde{C}_{33} = (137,265.35,460.35,732.3;0.2)$.

Step 5. An upper bound ($\tilde{Z}_U^*$) on the optimal value of the FFCTP ($\tilde{Z}_{FFCTP}^*$) by calculating the objective value of the initial feasible solution of the FFCTP, obtained in step 2, is as follows:

$\tilde{Z}_U^* = 10\tilde{c}_{11} + 5\tilde{c}_{12} + 20\tilde{c}_{22} + 5\tilde{c}_{32} + 10\tilde{c}_{33} + \tilde{f}_{11} + \tilde{f}_{12} + \tilde{f}_{22} + \tilde{f}_{32} + \tilde{f}_{33} = (145,276,481,761;0.2)$.

Therefore, the optimal value of the FFCTP must be between Z_L^* and $\tilde{Z}_U^*$ as follows:

$(137,265.35,460.35,732.3;0.2) \leq \tilde{Z}_{FFCTP}^* \leq (145,276,481,761;0.2)$.

7. Conclusions

This paper proposed a new method as the best approximation method, with representation both of the transportation cost and the fixed cost of the generalized trapezoidal fuzzy numbers. To this end, it found an approximation solution for the optimal solution to the fuzzy fixed-charge transportation problem. The lower and upper bounds on the fuzzy optimal value of the FFCTP can be easily obtained by using the best approximation method and this is the main advantage of the proposed method. The proposed method has been illustrated using a numerical example.

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