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Non-linear programming model proposal for mutual support distance of air defence systems

Salih Taşdemir[1,2,*] and Sibel Atan[1]

[1] Faculty of Economics and Administrative Sciences, Department of Econometrics, Haci Bayram Veli University, Ankara, Turkey
[2] Turkish Air Force Command, Ankara, Turkey

* Correspondence: salihtasdemirim@hotmail.com

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Abstract

With the advancements in technology, the deployment of air defence systems required for the defence of a country after procurement is considered as an important problem in terms of defence effectiveness. It is seen that criteria such as coverage, strike effectiveness and logistics network are taken into account while deploying. In addition, principles such as remote countermeasures, defence in depth and all-round defence are also taken into account. In this study, two different deployment models are proposed for point and area air defence. In this model, the optimum deployment distance at which other systems can support each other in addition to a deployed air defence system is determined. In the nonlinear target programming model proposed for point air defence, the number of missiles is considered to be reasonably balanced for perimeter defence, while for area defence, the nonlinear optimization problem is addressed by calculating the equally important coverage and strike effectiveness criteria values. In this study, it is observed that the approach based on mutual support distance has been used for the first time in the defence literature. According to the results obtained, it is observed that this proposed model for determining the mutual support distance solves the deployment problem optimally.

Keywords: mutual support distance of air defence systems, air defence systems features, deployment of air defence systems, non-linear programming.

1. Introduction

From the past to the present, the defence of the country is one of the most prioritised issues of all states in the world. In the last century, technological developments in the field of air vehicles have led to the development of air defence systems. The objective of air defence systems is to neutralise all types of manned and unmanned aerial vehi-cles, missiles, and ammunition that may originate from the air as Petrov (2019) pointed out. When systems are taken into inventory, performance measurements are made by taking into account a number of criteria. The decision as to the number of systems to be included in the inventory is informed by a number of factors, including the prevailing threat situation, the critical lo-cations to be protected, the size of the geographical area, the associated costs, the infrastructure and logistics capabil-ities. The optimal deployment of the systems under consideration is a distinct decision-making process as Air Force Doctrine Publication 3-01 Counterair Operations (2023), National Security Strategy of the United States of America (2017) noted.

A review of the literature reveals a multitude of studies investigating the optimal deployment location of air defence systems. A selection of these studies is presented below.

Leibowitz and Lieberman (1960) used a game theoretic approach to the selection and deployment of air defence systems with different performances. Ignizio and Shannon (1972) sought to identify the optimal coverage for the threat, taking into account the probable direction of attack. The issue was resolved by dividing the area to be defended into smaller units and assigning visi-bility values to these units. Ghose et al. (1993) developed a model for the missile battery placement problem using a dynamic programming approach. Based on the maximum probability of hitting a target criterion, the total hazard value is minimised. Franklin et al. (1994) developed a geometric algorithm that optimises the viewing angle in three dimensions for the deployment of air defence missile batteries.

In their study, Brown et al. (2005) presented an optimisation model in which the maximum expected total damage is minimised. The model was solved with equal weights and four criteria: the ratio of the target's priority status, the ratio of the target's vulnerability to the defence weapon, the target's re-engagement probability, and the threat ratio of the targets. The approach of taking the logarithm of the objective function was found to have no effect on the optimal solution. Consequently, the results were obtained by taking the logarithm of the sum of the criteria values.

Pant and Deep (2006) employed genetic algorithms to optimise the performance of air defence systems. The objec-tive was to maximise the expected survival sustainability with air attacks. The problem was analysed as a sin-gle-criteria problem with constraints such as those pertaining to personnel, weaponry, and so forth.

The Karasakal (2008) model considers two distinct problems: the optimal assignment of air defence systems on ships in a maritime task group to enemy air targets (the missile assignment problem) and the optimal placement of ships in designated areas over the sea (the zone assignment problem). In the context of the missile assignment problem, the objective is to maximize the probability of success for each defence system. Conversely, in the zone assignment problem, the objective is to maximize the total expected interdiction probability of the naval task group.

Tanergüçlü et al. (2012) developed a decision support system based on geographic information systems and loca-tion models for the optimal placement of air defence weapons and radars. Lötter and Vuuren (2016) devised a genetic algorithmic solution to a dynamic model of weapon assignment for an air defence system, incorporating three criteria: minimum total cost, maximum probability of hit and maximum re-engagement time.

Pietkiewicz et al. (2018a) optimised an air defence plan for a given area. Point air defence systems were tested on artificial sets of possible attack directions with different probabilities. The author proposed a heuristic method based on the objective function related to the evaluation criterion (maximum target visibility). Pietkiewicz et al (2018b) developed a two-dimensional heuristic algorithm to plan the positioning of air targets in a continuous area according to the criterion of maximum probability of destruction.

Özdemir (2019) presented a two-criteria (minimum cost, maximum target destruction) deployment model for re-gional air defence. In this study, a heuristic algorithm using particle swarm optimisation was developed. Taşdemir (2021) used a genetic algorithm for the deployment of different types and numbers of point air defence systems with three criteria (coverage, strike effectiveness, logistic distance). The results obtained with the convergence algorithm developed in this study were found to be compatible. Özyılmaz Çopur (2022) conducted a deployment study with the cluster coverage algorithm considering cost and range variables.

It can be seen that the studies in the military field such as Lee and Kwak (2008) and Lee and Baek (2005) generally deal with weapon-target assignment problems. Ac-cording to the literature review in this field, it is noteworthy that there are relatively few deployment studies. There is no study on mutual support distance, which is a different perspective of deployment.

In this study, a new optimal solution is proposed by establishing a non-linear programming model for the mutual support distance in the deployment of point and area air defence weapons.

2. Mutual Support Distance of Air Defence Systems

Air defence systems can be divided into two categories, point air defence systems and area air defence systems. While point air defence systems are created to ensure the protection of critical buildings, facilities, systems or infrastructure, area air defence systems are created to ensure the general defence of the country. The characteristics of point and area air defence systems created by expert opinion are shown in Table 1.

Table 1. Features of air defence systems

Point air defence systems	Area air defence systems
Low range	High range
Several systems are used together	It is used as a single
Low cost	Cost is high
It has many deployment alternatives	Deployment options are limited
It defends a single site with a large number of missiles or munitions	Defends a large geographical area with a small number of missiles
It provides all-round defence of equal importance	Gives more weight to the possible direction of the threat

When analysing the characteristics listed in Table 1, it can be seen that coverage and shooting efficiency are prominent in zone defence. Therefore, these two important issues have been taken into account in the pro-posed zone defence model. In addition, since the number of systems will be small, the mutual support distance between two systems is taken into account. Among the deployment alternatives, the location closest to the measured mutual support distance can be selected. For point air defence, due to the all-round defence, the large number of missiles and ammunition and the number of more than two systems, it is considered that the system can defend equally from all directions depending on the number of systems.

3. Methodology

The mutual support distance of air defence systems refers to the distance between the same or different types of systems. The mutual support distance [AB] between systems A-B is shown in Figure 1. This distance should be such that the areas of influence overlap to some extent. By overlapping the systems' areas of influence, it is possible to provide a balanced defence against threats of different levels.

It is generally accepted that the area of effect of air defence systems is circular. The radius of the systems, i.e. their range, can vary depending on the missile and sensor technology used. In point defence, where more than one air defence system is deployed together, it is considered sufficient for effective defence to ensure some overlap in the systems' area of effect. This allows coordinated defence against threats approaching at different altitudes. In this study, the mutual support distance of point and area air defence systems is examined separately.

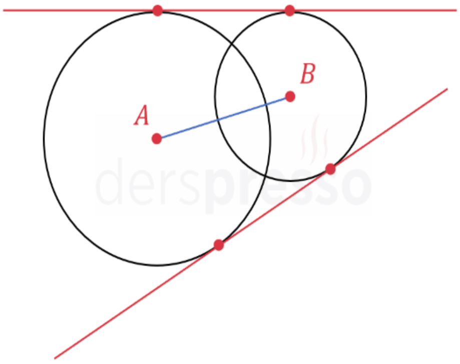

Figure 1. Representation of mutual support distance

The distance and location of the elements to be protected by point air defence systems should also be taken into account. The systems should be located sufficiently close to the elements to be protected, but at the same time they should be positioned to provide mutual support. Considering that resources are limited, the system should be able to provide support at an appropriate equilibrium point, depending on the number of units and the number of missiles available, so that it can provide equal defence. The parameters for the Non-Linear Programming (NLP) model, in which the same type of air defence systems support each other in n numbers and r ranges, are given below:

- r: Range of the same type of system in point air defence
- n: Number of units of the same system type in point air defence
- F: Number of missiles held by a system in point air defence

In area air defence, not only the defence of the defended area, but also the long-range air defence systems must be at a distance to protect and support each other. In order to support the closest long-range air defence system, the parameters for the non-linear programming (NLP) model of the problem, which is formed by using the equation of the area of intersection of two circles, are given below:

- $\mathrm{r_i}$: In area air defence, range of the i. system.
- $\mathrm{p_i}$: In area air defence, stroke efficiency of the i. system.

The variables included in the NLP models are, d: The distance between the deployment locations of two systems in the same line of defence (mutual support distance), where d is the sum of a and b, as shown in Figure 2.

$$b = \frac{r_1^2 - r_0^2 + d^2}{2*d} \tag{1}$$

$$a = \frac{r_0^2 - r_1^2 + d^2}{2*d} \tag{2}$$

Calculated Values h: As shown in Figure 2, it is half of the line formed by the intersection points of the coverage circles of the systems.

$$h^2 = r_1^2 - b^2 = r_0^2 - a^2 \tag{3}$$

$$h = \sqrt{r_1^2 - b^2} = \sqrt{r_1^2 - (\frac{r_1^2 - r_0^2 + d^2}{2*d})^2} \tag{4}$$

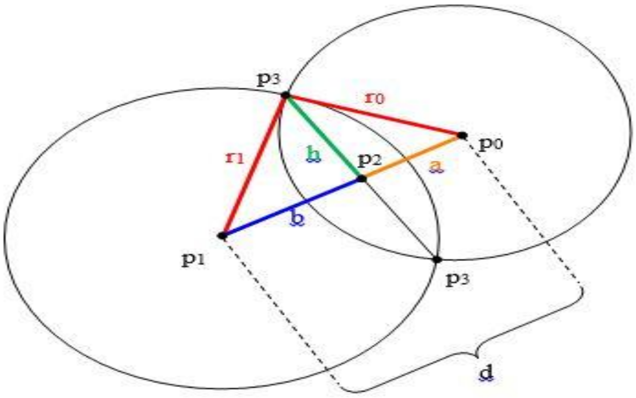

Figure 2. Notation for calculating the area of an intersection

KA: The area of intersection of two circles with area r_i is used for the average hit efficiency of the zone defence. As shown in Figure 2, the area of intersection of the two circles is constructed using the Pythagorean theorem.

$$\text{1st slice angle of the circle } \alpha = arccos\ \frac{a}{r_0} = arccos\ \frac{r_0^2 - r_1^2 + d^2}{2*d*r_0} \tag{5}$$

$$\text{1st slice angle of the circle } \beta = arccos\ \frac{b}{r_1} = arccos\ \frac{r_1^2 - r_0^2 + d^2}{2*d*r_1} \tag{6}$$

$$\text{Intersection Area (KA)} = 2*\left\{\left(\frac{\pi*r_1^2*\beta}{360} - \frac{b*h}{2}\right) + \left(\frac{\pi*r_0^2*\alpha}{360} - \frac{a*h}{2}\right)\right\} \tag{7}$$

$$\text{KA} = 2*\left\{\begin{array}{l} \left(\dfrac{\pi*r_1^2*arccos\,\frac{r_1^2-r_0^2+d^2}{2*d*r_1}}{360} - \dfrac{\left(\frac{r_1^2-r_0^2+d^2}{2*d}\right)*\sqrt{r_1^2-\left(\frac{r_1^2-r_0^2+d^2}{2*d}\right)^2}}{2}\right) + \\ \left(\dfrac{\pi*r_0^2*\ arccos\,\frac{r_0^2-r_1^2+d^2}{2*d*r_0}}{360} - \dfrac{(\frac{r_0^2-r_1^2+d^2}{2*d})*\sqrt{r_1^2-(\frac{r_1^2-r_0^2+d^2}{2*d})^2}}{2}\right) \end{array}\right\} \tag{8}$$

K: The area covered by the area air defence (total area covered) is the sum of the circles covered separately by the two air defence systems minus the intersection area.

P: The average strike effectiveness of the area covered by the air defence is obtained by multiplying the strike effectiveness of each area with different strike effectiveness and then dividing it by the total area.

$\mathrm{P_{min}}$: In the case of zone air defence, this is the average strike effectiveness in the case of maximum coverage, when the systems are tangent to each other and there is no intersection area.

$\mathrm{P_{max}}$: In area air defence, this is the average effectiveness at the lowest range when the circle of engagement of the lower-range system lies within the circle of engagement of the higher-range system and the intersection area is equal to the range of the lower-range system.

D: For point air defence, this is the evenly distributed number of F missiles supported by F missiles for all-round defence in the direction of the intersection point to provide equal defence.

Bringing air defence systems of the same type and in the same ring closer together increases the area of intersection, which increases the effectiveness of the attack. In the intersection area, two systems can engage the target at the same time and be more effective. However, coverage is also compromised. As the systems move away from each other, coverage increases and the effectiveness of the strike decreases. In addition, the intersection points of the circles move closer together, which can create a defensive vulnerability. This situation destroys the advantage of remote countermeasures.

4. Emprical Findings and Results

The mathematical model created to determine the mutual support distance of point and area air defence systems has been solved using two different approaches. These approaches are described below.

4.1 Mutual Support Distance of Point Air Defence Systems of the Same Type

The first step is to decide what the h-distance should be for the mutual support distance for point air defence systems, depending on the range. In this way, the total amount of missiles or ammunition to be fired as a result of the perimeter defence is balanced in other directions at an appropriate rate.

In this model, if the perimeter defence is $\frac{360}{n}$ degrees for n point air defence systems, it is expected to defend in all directions with a total of $n * F$ missiles. However, considering that possible future attacks will be launched from the position where the coverage and range is lowest, it is necessary to close the defence weakness at the intersections of the systems as much as possible without entering the intersection area. Therefore, two adjacent systems are expected to provide a total of $2 * (\frac{D}{2})$ missile support in proportion to the area between them. Therefore, if ammunition is shared in the interzones, the systems will be able to deploy as many missiles as D in their own direction so that F > D;

$$n * F = n * D + 2 * \frac{D}{2} \tag{9}$$

$$F = \frac{(n+1)*D}{n} \tag{10}$$

While interception can be done with number of missiles F at distance r, defence can be done with number of missiles D in attack time considering distance h with the ratio-proportion method

$$\frac{D}{F} = \frac{h}{r} \tag{11}$$

$$h = r * \frac{n}{n+1} \tag{12}$$

The mutual support distance corresponding to the distance h is found by the equation. The model is solved as a goal programming model. Since the ranges of the same type of systems are equal, equation 4 is set up as follows

$$h = \sqrt{r^2 - (\frac{d}{2})^2} = r * \frac{n}{n+1} \tag{13}$$

4.1.1 Constraints

The restrictive conditions in the model are given below. It is planned not to deploy the systems at the same point. This situation is represented by Equation 14.

$$d > 0 \tag{14}$$

The positions of the systems will be greater than the nearest distance h. This constraint is necessary to avoid negative values of a and b. Since d = a + b, it is not desirable for a or b to be negative.

$$\frac{r_0^2 - r_1^2 + d^2}{2 * d} = \frac{d}{2} \leq \mathrm{d} \tag{15}$$

At least one intersection is expected.

$$r_1 + r_0 = 2 * r \geq d \tag{16}$$

The solution results of the model are given in Table 2.

Table 2. The mutual support distance solution depends on the number of point air defence systems of the same type

R_1	R_0	n	d (km)
6	6	6	6.18
4	4	3	5.27
15	15	3	19.84
15	15	4	18
25	25	2	37.27

As the number of systems with the same range increases, the mutual support distance decreases. As the number of systems increases, the centre moves away from the point being defended. As the range increases, the mutual support distance increases.

4.2 Mutual Support Distance of Regional Air Defence Systems

The objective of this model is to find the mutual support distance between two equally weighted long-range area air defence systems with an optimum balance between average strike effectiveness and coverage. For area defence, it is sufficient to find the mutual support distance between two air defence systems. Since the number of long-range air defence systems will not be large, the main objective will be to deploy the minimum number of systems with maximum effectiveness. Since a new air defence system will be deployed in support of an existing deployed system, the other system will be considered fixed. Changing the location of the initial deployment will incur additional costs. Coverage and lethality are maximised.

$$Coverage\ (K) = \pi * r_1^2 + \pi * r_0^2 - KA \tag{17}$$

$$\text{Average Hit Efficiency } (P) = \frac{(|\pi * r_1^2 - KA|) * p_1 + KA * [1 - (1 - p_1) * (1 - p_0)] + (|\pi * r_0^2 - KA|) * p_0}{\pi * r_1^2 + \pi * r_0^2 - KA} \tag{18}$$

Normalised coverage assuming equal weighting and normalised average beat efficiency form the basis of our objective function.

4.2.1 Constraints

The constraints in the model are as follows. Systems must not be deployed at the same point.

$$d > 0 \tag{19}$$

The closest point at which each system supports the other is the point at which the system with less or equal range is fully covered.

$$|r_1 - r_0| \leq d \tag{20}$$

The deployment location of the systems must be greater than the distance closest to the line formed by the intersections of the coverage circles of the systems. This constraint is imposed to avoid negative values of a and b. Since d=a+b, it is not desirable for a or b to be negative.

$$0 < \frac{r_0^2 - r_1^2 + d^2}{2*d} \leq d \tag{21}$$

$$0 < \frac{r_1^2 - r_0^2 + d^2}{2*d} \leq d \tag{22}$$

At least one crossing is expected.

$$r_1 + r_0 \geq d \tag{23}$$

$$\mathrm{Z}_{\max} = 0.5 * \frac{\mathrm{K} - max\{\pi*\boldsymbol{r}_1^2,\ \pi*\boldsymbol{r}_0^2\}}{(\pi*\boldsymbol{r}_1^2 + \pi*\boldsymbol{r}_0^2) - max\{\pi*\boldsymbol{r}_1^2,\ \pi*\boldsymbol{r}_0^2\}} + 0.5 * \frac{P - P_{min}}{P_{max} - P_{min}} \tag{24}$$

The solution results of this model are given in Table 3.

Table 3. Equal weighted linear objective function solution results

R_1 (km)	R_0 (km)	P_1	P_0	Coverage (km^2)	Average Hit Efficiency	$\mathrm{Z}_{\max}$	d (km)
70	150	0.8	0.9	86079	0.8821	0.5*	220
70	150	0.8	0.9	70685	0.9174	0.5*	80
70	150	0.8	0.9	76736	0.9018	0.4761	132.6

In the equally weighted linear objective function model, the objective function is maximised when the mutual support distance can be the largest. The results of the solution are shown in Figure 3.

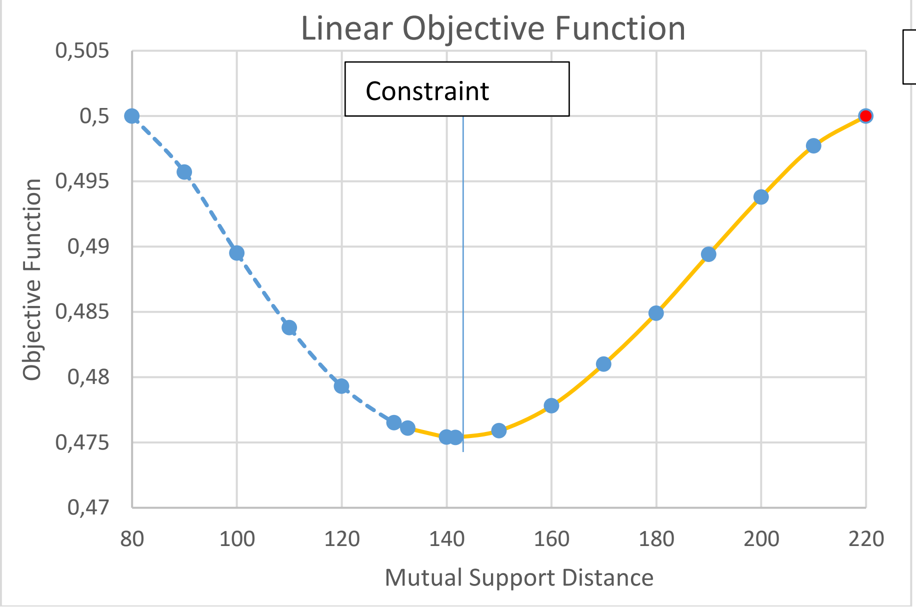

Figure 3. Equal weighted linear objective function

In order to find a suitable equilibrium point and to ensure that the criteria were equally effective, the equilibrium point was found by multiplying the normalised coverage by the normalised strike efficiency values in the objective function. The objective function is shown below.

$$\mathrm{Z_{max}} = \frac{\mathrm{K}-max\{\pi*\boldsymbol{r}_{\mathbf{1}}^{\mathbf{2}},\ \pi*\boldsymbol{r}_{\mathbf{0}}^{\mathbf{2}}\}}{(\pi*\boldsymbol{r}_{\mathbf{1}}^{\mathbf{2}}+\ \pi*\boldsymbol{r}_{\mathbf{0}}^{\mathbf{2}})-max\{\pi*\boldsymbol{r}_{\mathbf{1}}^{\mathbf{2}},\ \pi*\boldsymbol{r}_{\mathbf{0}}^{\mathbf{2}}\}} * \frac{P-P_{min}}{P_{max}-P_{min}} \tag{25}$$

Table 4 shows the solution results of the equally weighted multiplicative objective function.

Table 4. Equal weighted linear objective function solution results

R_1 (km)	R_0 (km)	P_1	P_0	Coverage (km^2)	Average Hit Efficiency	$\mathrm{Z_{max}}$	d (km)
70	150	0.8	0.9	78006	0.8989	0.225997	141.7
70	150	0.95	0.5	78006	0.5914	0.225997	141.7
70	150	0.5	0.95	78006	0.9103	0.225997	141.7
40	70	0.8	0.9	17730	0.8989	0.215981	63.8
40	70	0.95	0.5	17730	0.6313	0.215981	63.8
40	70	0.5	0.95	17730	0.8945	0.215981	63.8
25	25	0.95	0.95	2778	0.9696	0.171572	16.6
25	25	0.7	0.7	2778	0.7868	0.171572	16.6
25	25	0.5	0.5	2778	0.6034	0.171572	16.6

The effect of changing the systems' hit effectiveness values on the mutual support distance was not determined, but it did change the average hit effectiveness values. As the ranges of the systems increase, the value of the objective function increases. Therefore, the mutual support distance also increases. The graph of the change in the objective function for the result in the first row is shown in Figure 4.

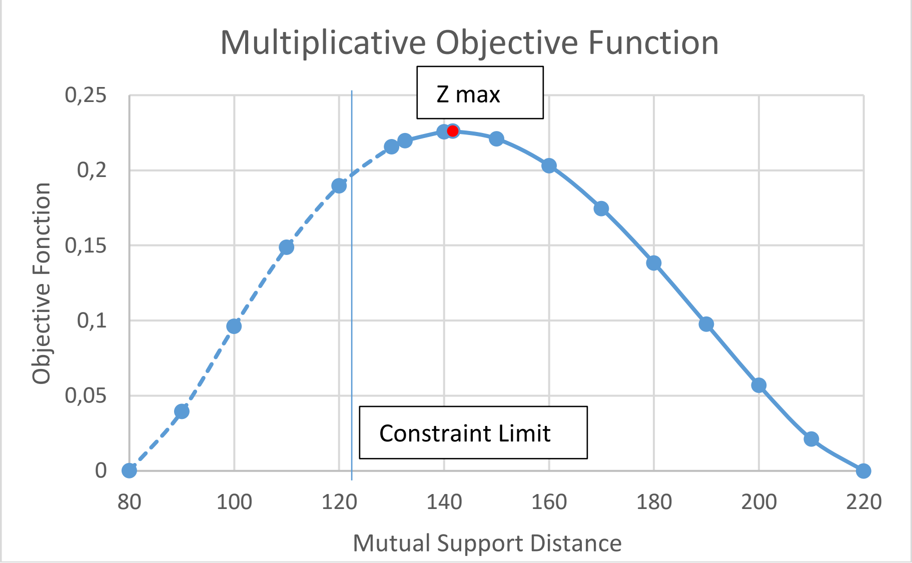

Figure 4. Equal weighted multiplicative objective function

It is possible to deploy systems with different strike effectiveness at the optimum mutual support distan-ce when the objective function is maximised. This is because the multiplicative objective function considers both criteria equally.

5. Conclusions and Discussion

Air defence systems are one of the most important elements of national security. In order to make the most efficient use of limited resources, a mathematical model has been developed of how to provide good protection with existing systems. While the mutual support distance of the same type of point air defence systems is found, the goal of equal defence of the existing missile or ammunition to provide all-round defence is achieved. A balanced defence planning model has thus been created. For long-range area air defence systems, taking into account the mutual support distance, the solution model was developed by finding the mutual support distance in the case where the normalised values of coverage and strike effectiveness of equal importance between 0-1 are equal. The studies in the literature propose the deployment model by considering all systems holistically. However, since it is sometimes possible to choose the location of new systems without changing the location of existing systems in terms of cost, this study was carried out. The proposed mutual support distance model, used for the first time in the literature, gave consistent results. In future studies, different models can be developed and compared with the models proposed in this paper.

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