

The presentation of the application of game theory to historical events

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Abstract

The modern world is characterized by a multitude of problems that contain conflicts of different interests. The importance and frequency of the problem caused the need to develop techniques for managing conflict situations. Accordingly, game theory has developed as a special direction and methodology used in the study, analysis and resolution of conflict situations. This theoretical direction deals with the phenomenon of making rational decisions in conflict situations and is considered a specific type of mathematical analysis of conflicts. It is a mathematical discipline that deals with the formalization of the decision-making process in situations where multiple decision-making subjects participate, who have conflicting interests. Each of the opposing sides has several strategies at their disposal. The goal is to determine the strategy that is best for each side in a conflict situation. The use of game theory provides a clearer representation of possible alternative solutions to a conflict situation. The paper shows the application of game theory to certain historical events. Through the petition payment matrix, solutions to the Battle of the Bismarck Sea, the Cuban Missile Crisis and the arms race are possible. It should be noted that for the analysis of a mathematical model, game theory requires quantifying and measuring something that is immeasurable, such as motives, outcomes, preferences, etc., when it comes to different (some) conflict situations. During the selection of the strategy, only the most important decision-making factors are included in the consideration, and the others are neglected, which can lead to the absence of a wider picture that could influence the final choice of strategy. Game theory is widely applicable, but the high degree of abstraction greatly limits its practical value.

Keywords: Game theory, Conflict situations, Nash equilibrium, Pareto optimality.

1. Introduction

The modern world is characterized by many problems, which contain conflicts of different interests (Biswas et al., 2024). The importance and frequency of the problem caused the need to develop techniques for managing conflict situations (Božanić et al., 2023). By studying the situations that are characteristic of the conflicts of the

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participants, it led to the formation of mathematical models of decision-making in conflict situations. The field of applied mathematics, which deals with the problems of conflict situations, is called game theory.

Game theory represents a special direction and methodology used in the study, analysis and resolution of conflict situations. This theoretical direction deals with the phenomenon of making rational decisions in conflict situations and is considered a specific type of mathematical analysis of conflicts. It is a mathematical discipline that deals with the formalization of the decision-making process in situations where multiple decision-making subjects participate, who have conflicting interests (Sahoo et al., 2024). One of the definitions of game theory is: "Game theory is a theory that deals with rational decision-making in conflict and partially conflict situations, when the interdependence of the actions of two or more participants determines individual outcomes" (Pavličić, 2010).

After World War II, game theory began to develop as a branch of economic science. However, game theory has a far wider application in other spheres of social life.

2. The concept of conflict and conflict situations

In theory, the concept of conflict is one of the controversial and unavoidable concepts, which is present in all societies. The theoretical narrative of the issue of conflict and conflict situations is very broad and widespread - from theoretical, biological, natural law, utopian and geopolitical, to various variants of psychological, sociopsychological and sociological theories, all the way to mathematical and game theories.

The etymological origin of the word conflict has its origin in the Latin language (Latin *conflictus*). Conflictologist, Lewis Coser (1956: 29) defines conflict as "a struggle for values and pretensions to a certain social status, power and material and spiritual goods, which are not enough for everyone, and where the goals of the opposing parties are: neutralization, damage or elimination of rivals." Conflict can also be defined as a disagreement between people, groups and others. Conflicts are and contain competition, opposites and incompatibilities of goals, values and interests. They express dissenting views and adversarial behavior and advocacy based on those views.

There is no universal definition of such a complex concept as conflict (Abid and Saqlain, 2024). In the literature, it is possible to find different definitions of the concept of conflict and conflict situations in which differences in relation to interests, goals, needs, ideas, etc. are emphasized, but all definitions have in common that a conflict situation is always between at least two parties.

The causes of conflicts can be diverse, complex and mutually conditioned (Djebara et al., 2024). They arise from different goals, interests and different social positions of groups, strata, classes, ethnic and religious communities, but also differences in values, personality, education, culture, expectations, etc. The root and basis of the conflict lies in conflicting interests. Interests are both a motivating factor and a driver of conflict, because individuals and groups take action to satisfy their interests (Jangid and Kumar, 2022). The subject of every conflict is a real or imagined problem, which serves as a reason or motive for confrontation (Shimoji, 2022). The object of the conflict consists of material and social values that the subjects in the conflict situation aspire to. The subjects of the conflict are the participants who have conflicting interests.

In order to talk about a conflict, there must be at least two subjects. The number of participants in the conflict is not limited, but it must be finite. The subjects are in mutual contact, they are aware of this contact and aware of the conflict of interests. If there is a disagreement or conflict of interest between the subjects of the conflict, but there is no awareness of it, the conflict will remain in a latent state.

2. Game theory

Game theory is a mathematical theory that studies a conflict situation characterized by a conflict of different interests (game), in which several opposing parties (players) participate, and each of these parties has at its

disposal several alternative solutions, i.e. possible moves (strategy) (Tadić et al., 2005). The player chooses the best strategy, i.e. the strategy that leads to a more favorable outcome of the game (Doğan and Esmerok, 2024).

2.1 Historical development of game theory

Back in ancient Greece, Plato and Socrates discussed war strategies in battles in their works. During the 17th century, the basic idea of game theory was to solve problems in various gambling games, which were played at French courts. Various aspects of game theory were at one time the subject of consideration by philosopher Thomas Hobbes, mathematician Blaise Pascal and others.

The first works in the field of game theory date back to the first half of the 19th century. In those works, Cournot and Bertrand hint at the possibility of using game theory for the purposes of economic analysis. The idea of the general theory of games and the basic terms were given by John von Neumann and Oskar Morgenstern in the book "Theory of Games and Economic Behaviour" (Vignjević, 2024). Also, this is the first comprehensive systematization of matter from game theory. In this book, a theory is written in two languages: mathematical symbolic language and descriptive, the language of economics (Neumann and Morgenstern, 1953). A big shift in the development of game theory was brought by John Nash (1951) with his capital work non-cooperative game. Nash (1951) introduced the notion of an equilibrium point, which is better known as the Nash equilibrium.

During the 50s and 60s of the 20th century, game theory models began to be applied in economic theory, the psychological study of people's behaviour in experimental games, in the sociological field of modelling election results, in political science where the concept of strategic voting appeared, etc. In the early 1970s, game theory appeared in the field of evolutionary biology, where the struggle for survival between species was modelled. In botany, with the help of game theory, the diffusion of seeds, the spread of roots in the ground is considered.

The mainstream of game theory cantered around the concept of Nash equilibrium. By 1987, human-subject game experiments provided the first illustration that laboratory human strategic behaviour could be accurately captured by this concept (Zhijan et al., 2022). Game theory has been widely applied in policy life making to achieve certain social and economic goals.

2.2 Basic terms in game theory

For a successful understanding of game theory, it is necessary to define and understand the basic terms, without which game theory could not exist. Basic terms in game theory are: game, player, strategy and game outcome.

Creators of modern game theory, John von Neumann and Oskar Morgenstern (1953), in the book "Theory of games and economic behavior", defined a game as the totality of rules that describe it, while the moves are the components of that game. A game can be said to be a strategic interaction between players. In other words, the term game means a conflict situation in which each participant only partially controls the situation. As a result of each player's choice of strategy, a certain state of the game is formed. The rules of the game are imposed instructions that each player must strictly adhere to. The term player means participants in a conflict situation. This term can mean one participant or a group of participants who take the same side in the conflict.

Strategy is defined as a set of possible solutions available to one player. The term "outcome" refers to the state that was created by the action of strategic decisions and moves of all players participating in the game. In other words, the outcome of the game is the result that ends the game. Most often, it is the profit made by one player, that is, the loss suffered by the other player.

The strategy of the game, for each player, represents a constant plan of action, during the entire game, as an opportunity to predict answers for every possible eventuality that may arise during the game (Kapor, 2017). The optimal strategy is a description of how the player could play the game in order to achieve the most favorable outcome for himself. A dominant strategy is one that is clearly best for some player, regardless of what the other player may play, and a dominated strategy is one that is clearly bad for some player, regardless of what the other

player plays. The strategic set represents the number of all actions that each player has when making decisions. The outcome of the game depends on the strategies of the players.

Each game consists of individual moves by the players, where a move represents one choice among possible alternatives. A strategy is one of the possible moves of a player, which is represented by a decision tree, and can be explained as a complete plan of options for each decision node, for each player.

Strategies are classified into pure and mixed. When a player consistently adheres to the choice of one strategy, then it is a pure strategy. The player can plan in advance how to run the game, from start to finish. In that case, the strategy must contain all possible cases that can happen during the game. At the very beginning, the player chooses a strategy and thus every alternative he undertakes during the game, regardless of the opponent's moves or some random events. A pure strategy represents the choice of a strategy with certainty, where the probability of its choice is equal to unity. A mixed strategy is a combination of a pure strategy and a strategy determined by a random selection process. When a player chooses his strategy without information about the moves and strategies of other players, then his strategy is reduced to a matter of simply choosing one of the offered alternatives, and is reduced to a game of probability. If the game is based on knowledge of the possible strategies of another player or players, then strategy dominates probability.

In game theory, the participants are the players. The number of players is at least two, and it is possible to have more players, with the fact that the number must be final. In the case of a larger number of players, it is possible to form a coalition, when members agree on strategies against other players. Players are placed in different situations, where they have to make decisions with the aim of getting the biggest payout (in any form). Each player should have more than one move available, except in the case of one-shot games. A greater number of moves develops a strategy, which affects the final outcome of the game.

Players are in conflict during the game. Their interests are conflicting, because all players want to win. The goal of game theory is to determine the optimal strategies for each participant in the game. Equilibrium (saddle) or equilibrium is the concept of a game solution, where it is assumed that all players know the equilibrium strategies of the other players and that no player can additionally achieve any gain or benefit by changing the strategy, i.e. no player he can profit by changing his strategy assuming that the other players do not change their strategies.

The concept of the so-called Nash equilibrium, which represents the concept of a game solution that involves each player making the best possible decision, taking into account the opponent's decision, which does not necessarily mean the biggest one, or total gain for all players. A Nash equilibrium can be represented as a set of strategies where neither player can do better if he unilaterally changes his strategy. The Pareto optimal situation is a situation in which no other possibility increases the profit of one player, with the condition that it does not decrease the profit of the other.

The absence of a saddle point means that if one player sets up a winning strategy, the other can always change his strategy and turn the loss into a win. A saddle point is stable, unlike a Nash equilibrium, because any deviation from the saddle point strategy reduces the payoff of each player.

3. Application of game theory in problem solving

Game theory began to develop after the Second World War, as part of the economic sciences. However, game theory has far wider applications. In addition to economic sciences, it can be equally successfully applied in other spheres and sciences such as: politics, international relations, law, religion, sports, philosophy, biology, social interactions and others. Game theory was applied in the search for answers to specific military and security issues.

In order to analyze conflict situations, with the help of a mathematical model, it is necessary to make a simplification, which allows the inclusion in the consideration of only the most significant factors that influence the possible outcome of the conflict. The goal of each player in the game is to achieve such a solution that ensures the achievement of the most favorable possible result. The magnitude of the results that individual players will

achieve in the game depends not only on their choice of strategies, but also on the choices of other players' strategies.

a) Battle of the Bismarck Sea

From March 2 to 4, 1943, the famous battle was fought in the Bismarck Archipelago. The United States of America, along with the Australian Air Force, fought against Japan. The US military received information about plans to move Japanese military forces from the port of Rabaul (eastern part of New Britain) to another port in New Guinea (Sweeney, 2020). The port in New Guinea can be reached either by the northern part of New Britain (a shorter route, but visibility is reduced and road conditions are worse) or by the southern part of New Britain (the route is much longer, but the road conditions are much better). Whichever route was chosen, the trip would take three days. American commanders were in a dilemma where to place their reconnaissance and aviation troops. If the convoy is discovered, it will be bombarded until it enters the port. If the Japanese convoys and American planes happen to be on different routes, it will take some time to detect the convoy. Table 1 shows the payment matrix, and the results in the matrix indicate the days of bombing.

Table 1. Payment matrix Battle of the Bismarck Sea

player I	player II	USA	
	strategy	North part	Southern part
Japan	North part	2, -2	2, -2
	Southern part	1, -1	3, -3

(Source: Stojanović, 2005)

From Table 1, it can be concluded that this is a game in which the gain of one player is equal to the loss of the other player. If the Japanese convoy moved in the northern part, and the American military forces were located in that line, it would take some time for the American military forces to detect the convoy (due to reduced visibility), and upon detection, they would bombard the Japanese convoy for two days. If the Japanese convoy were to move in the southern part, and the American military forces were located on the northern line, it would take time for the convoy to be discovered and the American military forces to move, so the Japanese convoy would be bombarded by the American forces (with reduced visibility) for the shortest period, which means least losses for the Japanese army. With the American forces on the southern line, and the Japanese convoy moving along the northern part, the Japanese would be bombarded for two days, with good visibility conditions. The worst possible scenario for the Japanese is that the convoy goes south, and American military forces are located on the southern line. Then the convoy would be bombarded for three days with excellent visibility conditions (Stojanović, 2005). The balance of this game is North-North, with the result of two days of low visibility bombing.

b) Cuban crisis

The Cuban crisis, from 1960, is one of the events in the sphere of international relations that can be presented using game theory. The crisis began on October 14, when US military aircraft took photos of Soviet warships transferring medium-range ballistic missiles to Cuba. Table 2 shows the payment matrix.

Table 2. Payment matrix for Cuban crisis

player I	player II	USSR	
	strategy	retreat	bring missiles
USA	do nothing	0, 0	-1, 1
	block the USSR	1, -1	-100, -100

(Source: Pavlović, 2015)

The USSR has deployed missiles and the US has the choice of doing nothing or blocking the USSR. If the US does nothing, bringing intermediate-range ballistic missiles to Cuba is a military and political achievement. In the event that the USA blockades the USSR, and the USSR withdraws as a result, the USA can be considered to have achieved slight superiority and demonstrated superiority. The most unfavorable situation for the entire planet would be a war, which would occur in a situation where the USA blocks the USSR, and the USSR defies and continues to bring missiles to Cuba. The most favorable situation is the withdrawal of the USSR without US intervention.

c) The arms race

Game theory can also be applied in international relations in the sphere of nuclear weapons. The first country to develop nuclear weapons has a comparative advantage. In the event that all countries develop nuclear weapons, no country has a comparative advantage anymore. For simplicity, only two countries (USA and USSR) will be shown in Table 3.

Table 3. Arms Race Payout

player I	player II		
	strategy	USSR	
		developed nuclear weapons	no nuclear weapons were developed
USA	developed nuclear weapons	1, 1	3, -1
	no nuclear weapons were developed	-1, 3	2, 2

(Source: Pavlović, 2015)

If both states decide not to develop nuclear weapons (2, 2), then the states are not in an arms race. In the event that the USSR chooses the non-development option, and the USA chooses the option of developing nuclear weapons (3, -1), then the USA comes into a position of superiority over the USSR. If both countries decide to develop nuclear weapons, they are in an arms race (1, 1). Finally, if the USSR chooses a strategy of development and the US chooses a strategy of non-development, the USSR enters a position of power (-1, 3). The best outcome for the USSR is (-1, 3), while the worst outcome is (3, -1).

From the table it can be concluded that it is best for all countries not to develop nuclear weapons. However, the equilibrium outcome of the game is (1, 1). For both sides in the game, this is a worse result, because $(1, 1) < (2, 2)$, but it is a better result than if they were in an asymmetric position (-1, 3) or (3, -1).

3.1 Model making

Creating (forming) a mathematical game model can be a very complicated task, since conflict situations are very diverse and complex. In the process of forming the model, it is important what information the players have. The goal of each player is to choose an optimal strategy, and it can be defined as the choice of a rational criterion for choosing a strategy in the light of actively identified, possible and probable actions of the opponent (Čupić and Tumala, 1991). When making a decision, the decision-maker must take into account both his goals and the goals of other participants, as well as their possible decisions (moves). Decisions will be made, such that the outcome of the game is the achievement of the goals of the decision-maker, even if this requires certain sacrifices in a certain phase of the game or deviation from maximalist requirements, including situations of mutual gain or minimal gain, if the development of the situation is unfavorable.

3.2. Solution of the problem

The resolution of the game is the outcome that ends the game, after all players have taken into account all the moves they could have made. In order for the player to reach the solution of the game, it is necessary to make a number of different decisions and go through the process of solving the problem. The problem solving process

consists of nine stages (Nikolić, 2012): observing the current situation and noticing problems; precise definition of the problem; defining goals (selection criteria); identification of alternative directions (alternatives, options); information gathering; evaluation of alternatives; choice of alternative; implementation of the action and analysis of the results.

The presented stages of problem solving, in practice, are rarely precisely defined and separated as separate units. In the course of the process, the decision maker may become aware that he must return to one of the previous stages in order to see the problem more complexly. It is possible that in the evaluation phase, the decision maker realizes that he cannot determine the optimal alternative based on the defined criteria, and in those situations he has to return to the previous phase. The time and effort that will need to be spent to solve the problem depends on the complexity of the problem, the importance of the outcome and the experience of the decision maker.

4. Conclusion

Conflict is present in all societies and represents a struggle between two or more entities that have opposing interests. In order to resolve conflict situations, it is necessary for the subjects in the conflict to make a number of different decisions.

Game theory is a mathematical theory that studies a conflict situation involving several opposing parties. Each of the opposing sides has several strategies at their disposal. The goal is to determine the strategy that is best for each side in a conflict situation.

The use of game theory provides a clearer representation of possible alternative solutions to a conflict situation. Based on mathematical and logical reasoning, the best strategy in the game is chosen. However, to analyze a mathematical model, game theory requires quantifying and measuring something that is immeasurable, such as motives, outcomes, preferences, and more, when it comes to different (some) conflict situations. During the selection of the strategy, only the most important decision-making factors are included in the consideration, and the others are ignored, which can lead to the absence of a broader picture that could influence the final choice of strategy. Game theory is widely applicable, but the high degree of abstraction greatly limits its practical value.

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