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An approach to multi-attribute decision-making based on intuitionistic fuzzy soft information and Aczel-Alsina operational laws

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Abstract

The intuitionistic fuzzy soft set (IFSS) is a vital technique for tackling uncertainty while the collection of information with the help of the membership function having values from unit interval. Moreover, the Aczel-Alsina t-norm (AATNRM) and Aczel-Alsina t-conorm (AATCRM) are the most generalized and flexible operational laws to operate the information which is the part of the unit intervals. The purpose of this article is to provide a number of aggregation operations (AOs) for information represented by intuitionistic fuzzy soft values (IFSVs) based on AATRM and AATCRM. Therefore, some new operational laws are developed by using on the AATRM and AATCRM for the development of the sum and product laws for IFSVs. Then, intuitionistic fuzzy soft Aczel-Alsina weighted averaging (IFSAAWA) and geometric (IFSAAWG) operators are purposed based on these operational laws. Additionally, some of their characteristics are examined, and the difference of the proposed and existing operators is investigated. Moreover, the proposed approach is applied to the problem of multi-attribute decision-making (MADM) for significance.

Keywords: The intuitionistic fuzzy soft set (IFSS), Aczel–Alsina aggregation operators, Multi-attribute decision-making (MADM)

1. Introduction

In modeling the real-life scenario to get the data, decision-making (DM) is rife with uncertainties. In order to solve the issue, several models have been developed. With the aid of the membership degree (MD), which contains values from a unit interval, Zadeh (1965) proposed the fuzzy set (FS) model to simulate real-life circumstances. FS quickly gained a lot of traction among scholars. Yet, FS only adequately reflected the actual circumstance. In order to address this issue, Atanassov (1986) suggested a new model called intuitionistic FS (IFS) with a non-membership degree (NMD) as an additional degree. The soft set (SS) model was developed by Molodtsov (1999) as an additional method to lessen the ambiguity and uncertainty of the information. Including only parameters based on MD allowed SS to generalize the FS. The addition of NMD in Maji et al. (2001) introduces the intuitionistic fuzzy SS (IFSS). The requirement stated in Maji et al. (2001) was that the MD and NMD sums for all parameters have to fall within the range of 0 and 1.

MADM is an excellent method for analyzing the list of options in terms of their qualities. Many academics have attempted to apply the MADM technique utilizing various fuzzy settings. AOs are essential to the MADM because they thoroughly aggregate the information (Garg and Rani, 2019a). Several practical methods for information aggregation have been developed as a result of the AOs. For instance, Lu et al. (2019), Khan et al. (2021), and Zhao et al. (2021), etc. introduced a variety of important AOs. These methods thoroughly aggregate the data of alternatives, as opposed to traditional AOs. The AOs have recently been recognized as the most fascinating field of research in MADM due to their broad variety of applications. A few examples of AOs developed in the recent few years are those by (Garg and Rani, 2019b), Deschrijver and Kerre (2002), Garg and Kumar (2018), and Garg and Arora (2018), as well as others in different fuzzy frameworks. You can find various techniques for treating the MADM in Liu et al (2021, 2022a, 2022b, 2023). Similar to Ali et al. (2021a), Arora and Garg (2017), and Hayat et al. (2021), who used complicated IFSS to build the AOs to solve the MADM problem, IFSS was used to develop AOs in each of these studies as well.

In fuzzy frameworks, information fusion is crucial. There are several operational laws that have been introduced for this reason. Operational laws like Frank's (Frank, 1979), Schweizer-Sklar's (Schweizer and Sklar, 1960) etc. On the basis of these operational laws, several academics introduced the AOs. Examples include Ali et al. (2021b) who used the t-norm (TRM) and t-conorm (TCRM) in power AOs, Seikh and Mandal (2021) who used the Dombi TRM and TCRM, Huang (2014) who used the Hamacher TRM, and Wang and Liu (2012) and Zhao and Wei (2013). Using Hamacher TRM and TCRM, Akram et al. (2021) presented the AOs for the complex IFS. In order to implement the AOs for IFS and address the MADM issues, Gao (2018) used Hamacher TRM and TCRM. AATRM and AATCRM are also introduced by Aczél and Alsina (1982) for the information fusion with greater flexibility. The AOs to address the MADM difficulties were developed by a number of writers using the AATRM and AATCRM. In order to address the MADM issue, Senapati et al. (2022) introduced the AOs based on AATRM and AATCRM for IFS. Hussain et al. (2022) utilized the generated AOs to address the MADM problem using PyFS and AATRM and AATCRM. Mahmood and Ali (2023) created the AOs to address the MADM problem using AATRM and AATCRM. Albaity et al. (2023) introduce the AOs for the PyFSS based on AATRM and AATCRM. Furthermore, a comparison by Farahbod and Eftekhari (2012) found that AATRM and AATCRM are the most adaptable and inclusive forms of operational regulations.

The major goal of this article is to develop some averaging and geometric operators based on AATRM and AATCRM to obtain the flexibility. The section 2 consists of some basic concepts, section 3 consists of the formulization of the IFSAAWA and IFSAAWG operators based on AATRM and AATCRM, section 4 consists of the applications of the proposed operators, and section 5 consists of conclusion of the study.

2. Preliminaries

Some fundamental ideas that were helpful in the creation of this work.

Definition 1. (Zadeh, 1965) A mapping $F: \varepsilon \rightarrow \mathbb{H}^{\mho}$ is known as a SS, where $\mathbb{H}^{\mho}$ be a family of all subsets of a universe of discourse $\mho$ and ε is a set of attributes.

Definition 2. Atanassov (1986) $\mathbb{H}^{\mho}$ be a family of all fuzzy subsets over U and ε be a set of attributes. Let $\mho\varepsilon$, then a pair $(F, \mho)$ is called FSS over U. Where F is a mapping such as $F: \mho \rightarrow \mathbb{H}^{\mho}$.

Definition 3. A mapping F: U → $I\mathbb{H}^{\mho}$ is known as an IFSS and defined as F$u_u(e) = \{(u_u,\ T_{\mho}(u_u)|\ u_u \in \mho\}$ where $T_{\mho}(u_u)\ and\ T_{\mho}(u_u)$ are the MD and NMD respectively for all $u_u \in \mho\ and\ 0 \leq T_{\mho}(u_{u)}, T_u(u_u), T_{\mho}(u_u) + T_{\mho}(u_u) \leq 1$. Where $I\mathbb{H}^{\mho}$ be a set of all intuitionistic fuzzy subsets of $\mho$.

Definition 4. Let $(F, \mho)$ and (G, B) be two IFSSs over $\mho$. Then some basic operations under IFSS defined as follows:

I $\mho$ $us\ subset\ of\ \beta\ and\ T_{F(e)} \leq T_{G(e)}\ and\ T_{F(e)} \geq T_{G(e)}\ \ for\ all\ e \in \mho\ , then\ (F, \mho) \subseteq (G\ , B)$.
If (F, U) ⊆ (G, B) and (G, B) is subset of $(F, \mho)$ then $F(\mho) = (G, B)$.
Let $F(\mho) = \{(u_u,\ T_{\mho}(u_u),\ T_{\mho}(u_u)|u_u \in \mho\}$.
Definition 5. For an IFSV $\beta_{uj} = (\varsigma_{uj}, \rho_{uj})$ score function is defined as:

$$S(\beta_{uj}) = \varsigma_{uj} - \rho_{uj} \tag{1}$$

Definition 6: The AATRM is the mapping from $[0,1] \times [0,1]$ to $[0,1]$ such that

$$\mathcal{F}(\mho, \mathbb{Q}) = \begin{cases} \mathcal{F}c(\mho, \mathbb{Q}) & if\ g = 0 \\ \max(\mho, \mathbb{Q}) & if\ g \to \infty \\ 1 - e^{-((ln\mho^{g}) + (-ln\mathbb{Q})^{g})^{1/g}} & otherwise \end{cases}$$

The AATCRM is the mapping from $[0,1] \times [0,1]$ to $[0,1]$ such that

$$Ⅎ(\mho, \mathbb{Q}) = \begin{cases} Ⅎ(\mho, \mathbb{Q}) & if\ g = 0 \\ \max(\mho, \mathbb{Q}) & if\ g \to \infty \\ 1 - e^{-((\ln(1-\mho)^{g}) + (ln(1-\mathbb{Q})^{g})^{1/g}} & otherwise \end{cases}$$

3. Proposed Aggregation Operators

This section contains the development of the proposed AOs based on the AATRM and AATCRM. IFSAAWA and IFSAAWG operators for the aggregation of the family of various IFSVs are presented in this section.

Definition 7. For the family of IFSVs $\beta_{uj} = (\varsigma_{uj}, \rho_{uj}); u = 1, 2, \ldots, n, j = 1, 2, \ldots, m$. Then IFSAAWA operator is created as follows.

$$IFSAAWA(\beta_{11}, \beta_{12}, \ldots, \beta_{nm}) = \oplus_{j=1}^{m} \zeta_j(\oplus_{u=1}^{n} \varrho_u \beta_{uj})$$

Where, the weight vectors are denoted by $\zeta_j\ , \varrho_u > 0, \sum_{j=1}^{m} \zeta_j = 1$ and $\sum_{u=1}^{n} \varrho_u = 1$.

Theorem 1. IFSAAWA operator yields the aggregated value of the collection of IFSVs, which is still an IFSV and is given by:

$$IFSAAWA(\beta_{11}, \beta_{12}, \ldots, \beta_{nm}) = \left(1 - e^{-\left(\sum_{j=1}^{m} \zeta_j\left(\sum_{u=1}^{n} \varrho_u(-ln(1-\varsigma_{uj}))^{\psi}\right)\right)^{\frac{1}{\psi}}}, e^{-\left(\sum_{j=1}^{m} \zeta_j\left(\sum_{u=1}^{n} \varrho_u(-ln(\rho_{uj}))^{\psi}\right)\right)^{\frac{1}{\psi}}}\right)$$

(2)

Proof. Let $\beta_{uj} = (\varsigma_{uj}, \rho_{uj})$ be a collection of IFSVs. Then, utilizing IFSV operating laws, we have

$$\varrho_u \beta = \left(1 - e^{-(\varrho_u(-ln(1-\varsigma))^{\psi})^{\frac{1}{\psi}}}, e^{-(\varrho_u(-ln(\rho))^{\psi})^{\frac{1}{\psi}}}\right)$$

And

$$\oplus_{u=1}^{n} \varrho_u \beta_{uj} = \left(1 - e^{-\left(\sum_{u=1}^{n} \varrho_u(-ln(1-\varsigma_{uj}))^{\psi}\right)^{\frac{1}{\psi}}}, e^{-\left(\sum_{u=1}^{n} \varrho_u(-ln(\rho_{uj}))^{\psi}\right)^{\frac{1}{\psi}}}\right)$$

Also, we have

$$\oplus_{j=1}^{m} \zeta_j(\oplus_{u=1}^{n} \varrho_u \beta_{uj}) = \left(1 - e^{-\left(\sum_{j=1}^{m} \zeta_j\left(\sum_{u=1}^{n} \varrho_u(-\ln(1-\varsigma_{uj}))^{\psi}\right)\right)^{\frac{1}{\psi}}}, e^{-\left(\sum_{j=1}^{m} \zeta_j\left(\sum_{u=1}^{n} \varrho_u(-\ln(\rho_{uj}))^{\psi}\right)\right)^{\frac{1}{\psi}}}\right)$$

Therefore,

$$IFSAAWA(\beta_{11},\beta_{12},...,\beta_{nm}) = \left(1-e^{-\left(\sum_{j=1}^{m}\zeta_j\left(\sum_{u=1}^{n}\varrho_u\left(-\ln\left(1-\varsigma_{uj}\right)\right)^{\psi}\right)\right)^{\frac{1}{\psi}}}, e^{-\left(\sum_{j=1}^{m}\zeta_j\left(\sum_{u=1}^{n}\varrho_u\left(-\ln\left(\rho_{uj}\right)\right)^{\psi}\right)\right)^{\frac{1}{\psi}}}\right)$$

Some basic properties of the proposed IFSAAWA operator are stated as follows.

Theorem 2. If $\beta_{uj}, \delta_{uj}; u = 1,2,\dots,n, j = 1,2,\dots,m$, are IFSVs. Then some properties are stated as follows.

- Idempotency: If all the IFSVs are identical, $u.e.,\beta uj = \beta$ for all u, j, then
$$IFSAAWA(\beta_{11},\beta_{12},...,\beta_{nm}) = \beta$$
- Monotonicity: Let $\beta_{uj} = (\varsigma_{uj},\rho_{uj})$ and $\delta_{uj} = (\tau_{uj},\upsilon_{uj})$ be two collections of the IFSVs such that $\varsigma_{uj} \leq \tau_{uj}$ and $\rho_{uj} \geq \upsilon_{uj}$ then $IFSAAWA(\beta_{11},\beta_{12},...,\beta_{nm}) \leq IFSAAWA(\delta_{11},\delta_{12},...,\delta_{nm})$.
- Boundedness: Let $\beta^- = \left(\min_j \min_u\{\varsigma_{uj}\}, \max_j \max_u\{\rho_{uj}\}\right)$ and $\beta^+ = \left(\max_j \max_u\{\varsigma_{uj}\}, \min_j \min_u\{\rho_{uj}\}\right)$. Then $\beta^- \leq IFSAAWA(\beta_{11},\beta_{12},...,\beta_{nm}) \leq \beta^+$.

Proof. Idempotency: If all β_{uj} are same i.e., $\beta_{uj} = \beta = (\varsigma,\rho), \forall\, u, j$ then by using Eq. 2, we have

$$IFSAAWA(\beta_{11},\beta_{12},...,\beta_{nm}) = \left(1-e^{-\left(\sum_{j=1}^{m}\zeta_j\left(\sum_{u=1}^{n}\varrho_u\left(-\ln\left(1-\varsigma_{uj}\right)\right)^{\psi}\right)\right)^{\frac{1}{\psi}}}, e^{-\left(\sum_{j=1}^{m}\zeta_j\left(\sum_{u=1}^{n}\varrho_u\left(-\ln\left(\rho_{uj}\right)\right)^{\psi}\right)\right)^{\frac{1}{\psi}}}\right)$$
$$= \left(1-e^{-(-\ln(1-\varsigma))}, e^{-(-\ln(\rho))}\right) = (\varsigma,\rho).$$

Proof. Monotonicity: Since we have $\varsigma_{uj} \leq \tau_{uj}$, therefore, for all u, j, we have

$$\sum_{u=1}^{n}\varrho_u\left(-\ln\left(1-\varsigma_{uj}\right)\right)^{\psi} \geq \sum_{u=1}^{n}\varrho_u\left(-\ln\left(1-\tau_{uj}\right)\right)^{\psi}$$

In the same way, we get

$$e^{-\left(\sum_{j=1}^{m}\zeta_j\left(\sum_{u=1}^{n}\varrho_u\left(-\ln\left(1-\varsigma_{uj}\right)\right)^{\psi}\right)\right)^{\frac{1}{\psi}}} \geq e^{-\left(\sum_{j=1}^{m}\zeta_j\left(\sum_{u=1}^{n}\varrho_u\left(-\ln\left(1-\tau_{uj}\right)\right)^{\psi}\right)\right)^{\frac{1}{\psi}}}$$

Similarly, we have

$$1-e^{-\left(\sum_{j=1}^{m}\zeta_j\left(\sum_{u=1}^{n}\varrho_u\left(-\ln\left(1-\varsigma_{uj}\right)\right)^{\psi}\right)\right)^{\frac{1}{\psi}}} \leq 1-e^{-\left(\sum_{j=1}^{m}\zeta_j\left(\sum_{u=1}^{n}\varrho_u\left(-\ln\left(1-\tau_{uj}\right)\right)^{\psi}\right)\right)^{\frac{1}{\psi}}}$$

Similarly, we can prove for the NMD. Hence, the proof is completed.

Proof. Boundedness: Since for all u, j we have $\beta^- \leq \beta_{uj} \leq \beta^+$, therefore by using monotonicity property, we get $\beta^- \leq IFSAAWA(\beta_{11},\beta_{12},\dots,\beta_{nm}) \leq \beta^+$.

Definition 8: For the family of IFSVs $\beta_{uj};\ u = 1,2,\dots,n, j = 1,2,\dots,m$. Then IFSAAWG operator is created as follows.

$$IFSAAWG(\beta_{11},\beta_{12},\dots,\beta_{nm}) = \otimes_{j=1}^{m}\left(\otimes_{u=1}^{n}\beta_{uj}^{\varrho_u}\right)^{\zeta_j}$$

Where, the weights of the alternatives and the attributes are $\zeta_j, \varrho_u > 0, \sum_{j=1}^{m}\zeta_j = 1$ and $\sum_{u=1}^{n}\varrho_u = 1$.

Theorem 3. IFSAAWG operator yields the aggregated value of IFSVs, which is still an IFSV and is given by:

$$IFSAAWG(\beta_{11},\beta_{12},\dots,\beta_{nm}) = \otimes_{j=1}^{m}\left(\otimes_{u=1}^{n}\beta_{uj}^{\varrho_u}\right)^{\zeta_j}$$
$$= \left(e^{-(\sum_{j=1}^{m}\zeta_j\left(\sum_{u=1}^{n}\varrho u\left(-\ln\varsigma_{uj}\right)^{\psi}\right)^{\frac{1}{\psi}}}, 1-e^{-(\sum_{j=1}^{m}\zeta_j\left(\sum_{u=1}^{n}\varrho u\left(-\ln\left(1-\rho_{uj}\right)\right)^{\psi}\right))^{\frac{1}{\psi}}}\right)$$

Similar to the IFSAAWA operator, the IFSAAWG operator has same properties. However, some basic properties of the IFSAAWG operator are provided in the following.

Theorem 4. If $\beta_{uj}, \delta_{uj}; u = 1,2,\dots,n, j = 1,2,\dots,m$, are IFSVs. Then we have

- Idempotency: If all the IFSVs are identical, $u.e.,\beta uj = \beta$ for all u, j, then

$$IFSAAWG(\beta_{11}, \beta_{12}, \ldots, \beta_{nm}) = \beta$$

- Monotonicity: Let $\beta_{uj} = (\varsigma_{uj}, \rho_{uj})$ and $\delta_{uj} = (\tau_{uj}, \upsilon_{uj})$ be two collections of the IFSVs such that $\varsigma_{uj} \leq \tau_{uj}$ and $\rho_{uj} \geq \upsilon_{uj}$ then $IFSAAWG(\beta_{11}, \beta_{12}, \ldots, \beta_{nm}) \leq IFSAAWG(\delta_{11}, \delta_{12}, \ldots, \delta_{nm})$.
- Boundedness: Let $\beta^- = \left(\min_j \min_u \{\varsigma_{uj}\}, \max_j \max_u \{\rho_{uj}\}\right)$ and $\beta^+ = \left(\max_j \max_u \{\varsigma_{uj}\}, \min_j \min_u \{\rho_{uj}\}\right)$. Then $\beta^- \leq IFSAAWG(\beta_{11}, \beta_{12}, \ldots, \beta_{nm}) \leq \beta^+$.

4. Application of the Proposed Approach to MADM

In this section, a method for resolving MADM problems using proposed AOs in an IFSS context has been described and demonstrated with an example.

4.1. Methodology

Suppose a set of p candidates $\mathfrak{F} = \{\tilde{\mathfrak{F}}^{(1)}, \tilde{\mathfrak{F}}^{(2)}, \tilde{\mathfrak{F}}^{(3)}, \ldots, \tilde{\mathfrak{F}}^{(p)}\}$, which are evaluated by group of experts $V = \{\mathfrak{Z}_1, \mathfrak{Z}_2, \ldots, \mathfrak{Z}_n\}$ over certain parameters $C = \{\mathfrak{U}_1, \mathfrak{U}_2, \ldots, \mathfrak{U}_m\}$. Evaluation given by an expert $\mathfrak{Z}_u, (u = 1, 2, 3, \ldots, n)$ on the parameter $\mathfrak{U}_j, (1, 2, 3, \ldots, m)$ are displayed as IFSVs and defined as $\beta_{uj} = [\varsigma_{uj}, \rho_{uj}]$ such that $\varsigma_{uj} \geq 0, \rho_{uj} \leq 1$ and $\varsigma_{uj} + \rho_{uj} \leq 1$. The highest candidate(s) are chosen using the suggested operators in the following stages.

1) Gather information on each elective $\mathfrak{B}^{(s)}$; $s = 1, 2, \ldots, p$ in terms of IFSVs $\beta_{uj}^{(s)}$ and their matrix is summarized as $\left(\beta_{uj}^{(s)}\right)_{n \times m}$.
2) Aggregate the rating values $g_{uj}^{(s)}$ of each candidate $\mathfrak{B}^{(s)}$ into an aggregated one $g^{(s)}$ by IFSAAWA operator given in equation 1.
3) Find the score value of $g^{(s)}; s = 1, 2, \ldots, p$ using Eq. 1.
4) Rank the candidates $\mathfrak{B}^{(s)}(s = 1, 2, \ldots, p)$ by using the ascending value $S(g^{(s)})$ and hence find the best one(s).
5) Finish.

Example:

Using the recruitment of a lecturer in the Physics Department for a Government University as an example based on some attributes is an interesting example of MADM. A group of five experts $\{\mathfrak{Z}_1, \mathfrak{Z}_2, \mathfrak{Z}_3, \mathfrak{Z}_4, \mathfrak{Z}_5\}$ with weight vector $(0.3, 0.22, 0.18, 0.17, 0.13)$ will judge four applicants $\mathfrak{F}^{(1)}, \mathfrak{F}^{(2)}, \mathfrak{F}^{(3)}, \mathfrak{F}^{(4)}$ and will choose the best eligible candidates based on a certain parameter. C= $\{\mathfrak{U}_1, \mathfrak{U}_2, \mathfrak{U}_3, \mathfrak{U}_3, \mathfrak{U}_4, \mathfrak{U}_5\}$ which represent "Qualification", "Teaching experience", "Ability", "Research experience", "Publications" respectively with weights (0.25, 0.15, 0.3, 0.1, 0.2). To select best candidate, follow the steps given below:

1) Each expert assigns a rating to each candidate based on the IFSVs for each parameter which are provided in Tables 1, 2, 3 and 4.

Table 1. For candidate $\tilde{\mathfrak{F}}^{(1)}$

	$\mathfrak{U}_1$	$\mathfrak{U}_2$	$\mathfrak{U}_3$	$\mathfrak{U}_4$	$\mathfrak{U}_5$
$\tilde{\mathfrak{Z}}_1$	(0.2, 0.7)	(0.1, 0.7)	(0.6, 0.2)	(0.2, 0.6)	(0.2, 0.5)
$\tilde{\mathfrak{Z}}_2$	(0.3, 0.4)	(0.5, 0.3)	(0.5, 0.3)	(0.2, 0.3)	(0.8, 0.1)
$\tilde{\mathfrak{Z}}_3$	(0.4, 0.3)	(0.4, 0.1)	(0.6, 0.3)	(0.4, 0.5)	(0.7, 0.2)
$\tilde{\mathfrak{Z}}_4$	(0.1, 0.6)	(0.7, 0.1)	(0.6, 0.2)	(0.1, 0.5)	(0.3, 0.4)
$\tilde{\mathfrak{Z}}_5$	(0.3, 0.5)	(0.4, 0.2)	(0.6, 0.2)	(0.7, 0.1)	(0.2, 0.1)

Table 2. For candidate $\tilde{\mathfrak{F}}^{(2)}$

	$\mathfrak{U}_1$	$\mathfrak{U}_2$	$\mathfrak{U}_3$	$\mathfrak{U}_4$	$\mathfrak{U}_5$
$\overline{\mathfrak{Z}_1}$	(0.2, 0.6)	(0.2, 0.4)	(0.4, 0.3)	(0.2, 0.4)	(0.3, 0.4)
$\overline{\mathfrak{Z}_2}$	(0.4, 0.6)	(0.5, 0.1)	(0.6, 0.1)	(0.1, 0.4)	(0.1, 0.2)
$\overline{\mathfrak{Z}_3}$	(0.4, 0.4)	(0.5, 0.2)	(0.4, 0.1)	(0.3, 0.4)	(0.6, 0.1)
$\overline{\mathfrak{Z}_4}$	(0.1, 0.7)	(0.5, 0.2)	(0.4, 0.3)	(0.3, 0.1)	(0.2, 0.5)
$\overline{\mathfrak{Z}_5}$	(0.2, 0.4)	(0.5, 0.3)	(0.7, 0.2)	(0.4, 0.2)	(0.4, 0.3)

Table 3. For candidate $\tilde{\mathfrak{F}}^{(3)}$

	$\mathfrak{U}_1$	$\mathfrak{U}_2$	$\mathfrak{U}_3$	$\mathfrak{U}_4$	$\mathfrak{U}_5$
$\overline{\mathfrak{Z}_1}$	(0.4, 0.3)	(0.5, 0.3)	(0.6, 0.2)	(0.6, 0.1)	(0.6, 0.1)
$\overline{\mathfrak{Z}_2}$	(0.5, 0.2)	(0.7, 0.2)	(0.7, 0.1)	(0.3, 0.5)	(0.6, 0.2)
$\overline{\mathfrak{Z}_3}$	(0.6, 0.2)	(0.8, 0.1)	(0.6, 0.2)	(0.5, 0.4)	(0.7, 0.1)
$\overline{\mathfrak{Z}_4}$	(0.8, 0.1)	(0.6, 0.2)	(0.8, 0.1)	(0.6, 0.3)	(0.6, 0.1)
$\overline{\mathfrak{Z}_5}$	(0.6, 0.3)	(0.7, 0.2)	(0.7, 0.2)	(0.6, 0.1)	(0.8, 0.1)

Table 4. For candidate $\tilde{\mathfrak{F}}^{(4)}$

	$\mathfrak{U}_1$	$\mathfrak{U}_2$	$\mathfrak{U}_3$	$\mathfrak{U}_4$	$\mathfrak{U}_5$
$\overline{\mathfrak{Z}_1}$	(0.7, 0.1)	(0.2, 0.6)	(0.1, 0.7)	(0.6, 0.2)	(0.1, 0.8)
$\overline{\mathfrak{Z}_2}$	(0.6, 0.2)	(0.1, 0.8)	(0.3, 0.5)	(0.5, 0.2)	(0.3, 0.6)
$\overline{\mathfrak{Z}_3}$	(0.5, 0.3)	(0.3, 0.6)	(0.2, 0.7)	(0.6, 0.3)	(0.1, 0.7)
$\overline{\mathfrak{Z}_4}$	(0.6, 0.3)	(0.2, 0.7)	(0.4, 0.6)	(0.5, 0.3)	(0.3, 0.6)
$\overline{\mathfrak{Z}_5}$	(0.7, 0.2)	(0.1, 0.6)	(0.3, 0.6)	(0.7, 0.2)	(0.2, 0.6)

2) The aggregated values of different candidates by utilizing IFSAAWA operator are:

$$\mathbb{H}^{(1)} = (0.4519, 0.2995);\ \mathbb{H}^{(2)} = (0.3767, 0.2669)$$
$$\mathbb{H}^{(3)} = [0.6385, 0.1593];\ \mathbb{H}^{(4)} = [0.3774, 0.4097]$$

3) The score values for every candidate by utilizing IFSAAWA operator are:

$$S(\mathbb{H}^{(1)}) = 0.1524, S(\mathbb{H}^{(2)}) = 0.1098, S(\mathbb{H}^{(3)}) = 0.4791, S(\mathbb{H}^{(4)}) = -0.0324$$

4) The candidates are ordered as $\tilde{\mathfrak{F}}^{(3)} \succ \tilde{\mathfrak{F}}^{(1)} \succ \tilde{\mathfrak{F}}^{(2)} \succ \tilde{\mathfrak{F}}^{(4)}$ by IFSAAWA operator. Hence the applicant $\mathfrak{B}^{(3)}$ is most desirable applicant.

5) The score values for every candidate by utilizing IFSAAWG operator are:

$$S(\mathbb{H}^{(1)}) = -0.03907, S(\mathbb{H}^{(2)}) = -0.051631478, S(\mathbb{H}^{(3)}) = 0.42534, S(\mathbb{H}^{(4)}) = -0.30832$$

The candidates are ordered as $\tilde{\mathfrak{F}}^{(3)} \succ \tilde{\mathfrak{F}}^{(1)} \succ \tilde{\mathfrak{F}}^{(2)} \succ \tilde{\mathfrak{F}}^{(4)}$ by IFSAAWG operator. Hence the applicant $\mathfrak{F}^{(3)}$ is most desirable applicant.

Hence, we may conclude that the candidate $\mathfrak{F}^{(3)}$ is the most suitable for the hiring according to IFSAAWG operator. The results obtained from the IFSAAWA and IFSAAWG operators are same. However, the IFSAAWG operator is considered as more reliable due to its nature. The IFSAAWA operator is based on the average mean of the information while the IFSAAWG operator is based on the geometric mean of the information.

The result obtained with IFSAAWG operator is also given in Table 5.

4.2. Comparative Studies

The acquired findings are contrasted with existing approach in the IFSS environment, such as the intuitionistic fuzzy soft weighted Einstein averaging (IFSWEA) (Arora, 2020) operator and intuitionistic fuzzy soft weighted Einstein averaging (IFSWEG) (Arora, 2020), to demonstrate the effectiveness of the intended task. Table 5 shows the outcomes. This table shows that the outcomes produced by existing techniques are identical to those of a targeted strategy. In this paper, evaluation information is combined using the two aggregation operators IFSAAWA and IFSAAWG, and the candidates are then rated using the score function.

Table 5. Comparative studies

Approach	Rating values				Ranking
	$\tilde{\mathfrak{F}}^{(1)}$	$\tilde{\mathfrak{F}}^{(2)}$	$\tilde{\mathfrak{F}}^{(3)}$	$\tilde{\mathfrak{F}}^{(4)}$	
Purposed IFSAAWA operator	0.15244	0.10983	0.47915	-0.03236	$\tilde{\mathfrak{F}}^{(3)} \succ \tilde{\mathfrak{F}}^{(1)} \succ \tilde{\mathfrak{F}}^{(2)} \succ \tilde{\mathfrak{F}}^{(4)}$
Purposed IFSAAWG operator	-0.03907	-0.05163	0.42534	-0.30832	$\tilde{\mathfrak{F}}^{(3)} \succ \tilde{\mathfrak{F}}^{(1)} \succ \tilde{\mathfrak{F}}^{(2)} \succ \tilde{\mathfrak{F}}^{(4)}$
IFSWEA Operator (Arora, 2020)	0.12627	0.08858	0.47645	-0.07717	$\tilde{\mathfrak{F}}^{(3)} \succ \tilde{\mathfrak{F}}^{(1)} \succ \tilde{\mathfrak{F}}^{(2)} \succ \tilde{\mathfrak{F}}^{(4)}$
IFSWEG Operator (Arora, 2020)	-0.01055	-0.02605	0.43290	-0.27683	$\tilde{\mathfrak{F}}^{(3)} \succ \tilde{\mathfrak{F}}^{(1)} \succ \tilde{\mathfrak{F}}^{(2)} \succ \tilde{\mathfrak{F}}^{(4)}$

Table 5 shows the results obtained from the different operators defined for the framework of the IFSS. It is cleared from Table 5 that the ranking results are identical. It highlights the importance of the proposed operator. However, the proposed operator is developed based on the AATRM and AATCRM which are the most flexible operational laws. Consequently, the suggested method offers more versatile outcomes than the current operators. The geometric representation of the comparative study is represented by Figure 1 as follows.

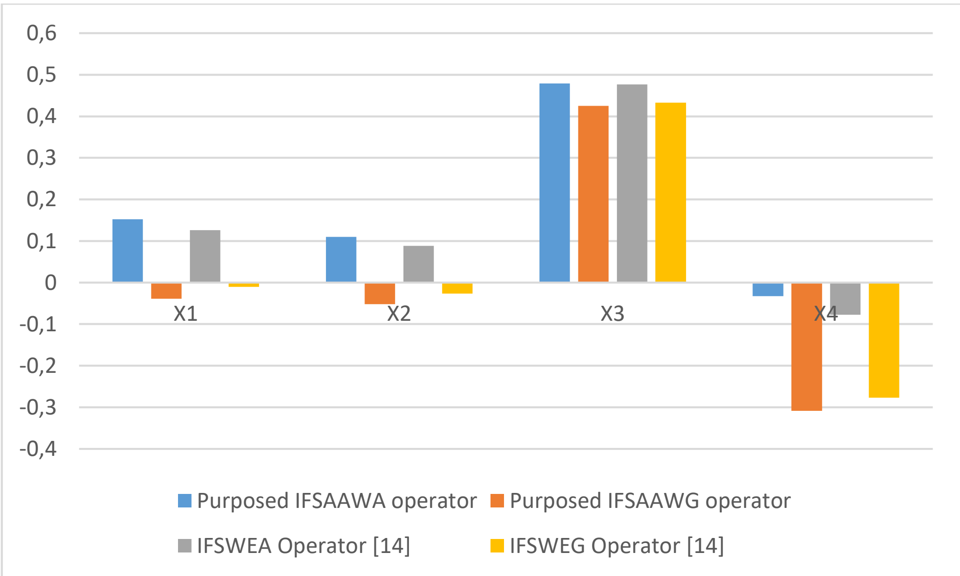

Figure 1. The geometrical representation of the comparative between different operators

5. Conclusion

In this study, we investigate DM issues with IFSS data, where a group of experts provides their preferences for each candidate based on a set of factors. IFSWAAWA and IFSAAWG, two geometric and averaging operators, are proposed based on these Aczel-Alsina operations. A few desired characteristics of these operators are also addressed. The last step is the provision of a DM strategy based on proposed operators. To illustrate the applicability and viability of the suggested approaches, an example has been provided. In order to demonstrate the efficacy of the suggested work, a contrast with some existing methodologies has also been made. In the proposed study, we presume that the properties of a particular set are independent of one another. The characteristics are typically reliant on one another in many realistic situations, though. The AATRM and AATCRM serve as the basis for the planned operators. The AATRM and AATCRM are the most flexible operational laws. Hence, the propose operators are the flexible operators for the aggregation of the information in the shape of the IFSVs. Therefore, the proposed work provides maximum flexibility to the decision makers. We aim to apply the same operational laws to the framework defined in Mahmood (2022), Riaz and Farid (2022), Riaz and Hashmi (2019) and frameworks defined in Akram and Bibi (2023) and Mahmood and Ur Rehman (2022).

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