Original Scientific Article Journal of Decision Analytics and Intelligent Computing
Vol. 4 issue 1, (2024) 1-15 https://doi.org/10.31181/jdaic10016012024p

A novel hybrid grey-BCM approach in multi-criteria decision making: An application in OTT platform

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Received 17 March 2023

Accepted for publication 8 January 2024

Published 16 January 2024

Abstract

Real world decision making is like a puzzle having complex, uncertain and vague information and this fact portray the wide range applicability of grey system theory in decision making procedure as grey system theory deals with the systems having information with uncertainty. In order to extend the base-criterion method to uncertain conditions, grey information may be a better way to solve a lot of multi-criteria decision-making problems. In this paper, we proposed a novel approach 'grey base-criterion method' (GBCM) based on the linguistic variables extended to the grey information. Weights of criteria have been calculated using GBCM. Numerical examples are illustrated and then the results are compared by the grey best-worst method (GBWM). Results of comparison show the high reliability of GBCM method with less consistency ratio over GBWM. A real case study of the fastest growing OTT (Over the Top) platforms in India has been taken to bestow the robustness of the proposed method.

Keywords: Grey system theory, Grey-base-criterion method, Group multi-criteria decision making, Pairwise-comparison, Over the Top (OTT) platform.

1. Introduction

Multi-Criteria Decision Making (MCDM) has been developed as an important part of operational research. In decision-making, solver must choose best solution from the given set of good solutions (Korhonen, 1992). The MCDM methods can be categorized into two main parts: first, to allocate the weights to all the criteria under consideration; second, by ranking alternatives based on their performance. Over the periods, many MCDM methods has been developed by researchers and decision makers. Methods including ranking of alternatives are, TOPSIS (Technique for the Order Preference by Similarity to Ideal Solution) (Hwang and Youn, 1981), VIKOR

(VlšeKriterijumska Optimizacija i Kompromisno Resenje) (Opricović, 1998), SWARA (Step-wise Weighted Assessment Ratio Analysis) (Keršuliene et al., 2010), PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) (Brans and De Smet, 2016) and ELECTRE (Elimination and Choice Expressing REality) (Kumar et al., 2017). Many MCDM methods involving allocation of weights to criteria are introduced as AHP (Analytic Hierarchy Process) (Satty, 1990), ANP (Analytic Network Process) (Saaty, 2004), BWM (Best Worst Method) (Rezaei, 2015), and BCM (Base- Criterion Method) (Haseli et al., 2020).

The BCM method was first introduced by Haseli et al. (2020), in which one criteria among different criteria selected as base criterion. Then the pairwise comparison between base-criterion and other criteria is done by using numerical scale from 1/9 to 9. Other pairwise comparison is done by using $a_{Base,i} * a_{i,j} = a_{Base,j.}$ further a min-max model is obtained to calculate the weights of criteria. Main advantages of BCM are that it includes less pairwise comparison and low inconsistency ratio which gives better solution for any real-world problem. This method has been used in many real world problems such as supplier selection (Ayough et al., 2023), and stock selection (Narang et al., 2021; Narang et al., 2022b).

It has been noticed that there is uncertainty and ambiguity in real world decision-making problems. So, it will be a right step to extend MCDM methods into uncertainty or poor information. Grey System Theory was given by Julong (1989). A grey system is made of partial known information and partial unknown information. Known information lies under white number and unknown information lies under black number. In between black and white numbers there are grey numbers. So, the concept of grey number has been used for incomplete information system. This fact inspires to introduce the grey system theory in several MCDM methods for solving real world problems (Datta et al., 2013; Kose et al., 2013; Bai and Sarkis, 2013; Kaviani et al., 2020). Fuzzy set theory (Zadeh, 1996) is also an approach to deal with uncertain and doubtful decision-making problems. Several tools have been proposed as interval-valued fuzzy set, type-2 fuzzy set, intuitionistic fuzzy set, linguistic fuzzy set and hesitant fuzzy set (Han and Trimi, 2018; Guo and Zhao, 2017; Mardani et al., 2015; Dožić et al., 2018; Narang et al., 2022a).

In the current study, the base-criterion method has been extended to grey information. The obtained results are compared with GBWM. The result calculated by GBCM is more reliable than the one calculated in GBWM (Mahmoudi et al., 2020). Computation with grey numbers is much easier than other uncertainty systems. After that the proposed approach has been employed to the OTT platforms to rank the alternatives. In the next section, the basic concepts of grey number system are discussed in detail.

2. Material and methods

2.1 Preliminaries

A number whose exact value is unknown but the range or interval in which it lies is known referred as grey number. A grey number can be expressed as $\otimes A \in [\underset{\sim}{a}, \tilde{a}]$, $\underset{\sim}{a} < \tilde{a}$, where $\underset{\sim}{a}$ is lower bound and $\tilde{a}$ is upper bound of grey number $\otimes$A. Length of grey number can be calculated as $\tilde{a} - \underset{\sim}{a}$.

2.1.1 Properties

A grey number have neither lower bound nor upper bound is called a black number, i.e., $\otimes A = [-\infty, +\infty]$, here $\otimes A$ is a black number.

A grey number have same lower bound as well as upper bound is called a white number, i.e., $\otimes A = [\underset{\sim}{a}, \tilde{a}]$, and $\underset{\sim}{a} = \tilde{a}$.

Core or kernel of a grey number is:

$$\otimes\hat{A} = \frac{1}{2}(\tilde{a} + \underset{\sim}{a})$$

If $\otimes A = [\underset{\sim}{a}, \tilde{a}]$ and $\otimes B = [\underset{\sim}{b}, \tilde{b}]$, the arithmetic operations on grey numbers are:

$\otimes A + \otimes B = [\underset{\sim}{a} + \underset{\sim}{b}, \tilde{a} + \tilde{b}]$,
$\otimes A - \otimes B = [\underset{\sim}{a} - \tilde{b}, \tilde{a} - \underset{\sim}{b}]$,
$\otimes A . \otimes B = \left[Min\{\underset{\sim}{a}\underset{\sim}{b}, \tilde{a}\tilde{b}, \tilde{a}\underset{\sim}{b}, \underset{\sim}{a}\tilde{b}\}, Max\{\underset{\sim}{a}\underset{\sim}{b}, \tilde{a}\tilde{b}, \tilde{a}\underset{\sim}{b}, \underset{\sim}{a}\tilde{b}\}\right]$,
$\frac{\otimes A}{\otimes B} = \otimes A . \otimes B^{-1} = \left[Min\left\{\frac{\underset{\sim}{a}}{\underset{\sim}{b}}, \frac{\underset{\sim}{a}}{\tilde{b}}, \frac{\tilde{a}}{\underset{\sim}{b}}, \frac{\tilde{a}}{b_2}\right\}, Max\left\{\frac{\underset{\sim}{a}}{\underset{\sim}{b}}, \frac{\underset{\sim}{a}}{\tilde{b}}, \frac{\tilde{a}}{\underset{\sim}{b}}, \frac{\tilde{a}}{\tilde{b}}\right\}\right]$,
$\otimes A^{-1} = \left[\frac{1}{\tilde{a}}, \frac{1}{\underset{\sim}{a}}\right]$
The grey possibility degree $P\{\otimes A \leq \otimes B\} = \frac{Max\{0, L(\otimes A) + L(\otimes B) - Max(0, \tilde{a} - \underset{\sim}{b})\}}{L(\otimes A) + L(\otimes B)}$

2.1.2 Grey linear programming

Linear programming is a mathematical model to get the best solution from the set of solutions. In real world problem, the input data suffers from a certain degree of uncertainty or incompleteness. So, this problem may handle with grey number system. If a decision maker is taking grey numbers as input data in any linear model then this is an example of grey linear programming.

Dang and Forrest (2009) presented a positioned programming method for solving GLP model which covers mainly all the uncertainties from the input which is grey number.

Positioned programming method for solving grey linear programming problem is:

$$
\begin{aligned}
&MaxS = C(\otimes)X \\
&s.t. \\
&A(\otimes)X \leq b(\otimes), \\
&X \geq 0. \\
&C(\otimes) = [C_1(\otimes), C_2(\otimes), \ldots\ldots, C_n(\otimes)]^T, \\
&b(\otimes) = [b_1(\otimes), b_2(\otimes), \ldots\ldots\ldots, b_m(\otimes)]^T, \\
&A(\otimes) = \begin{bmatrix} a_{11}(\otimes) & a_{12}(\otimes) & \ldots & a_{1n}(\otimes) \\ a_{21}(\otimes) & a_{22}(\otimes) & \ldots & a_{2n}(\otimes) \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1}(\otimes) & a_{m2}(\otimes) & \ldots & a_{mn}(\otimes) \end{bmatrix} \qquad (1)
\end{aligned}
$$

Where $A(\otimes)$is the grey consumption matrix, $C(\otimes)$ is the grey price vector, $b(\otimes)$is the grey constraint vector, and X is the problem decision vector.

$c_j(\otimes) \in [\underset{\sim}{c}, \tilde{c}],\ \underset{\sim}{c} \geq 0, j = 1,2, \ldots\ldots, n$
$b_i(\otimes) \in [\underset{\sim}{b}, \tilde{b}], \underset{\sim}{b} \geq 0, i = 1,2, \ldots\ldots, m,$
$a_{ij}(\otimes) \in [\underset{\sim}{a}, \tilde{a}], \underset{\sim}{a}_{ij} \geq 0, i = 1,2, \ldots., m, j = 1,2, \ldots., n.$

to solve the grey linear programming, whitenization of grey number is done.

Definition 1. White values of grey parameter can be determined as

$c'_j(\otimes) = \rho_j\tilde{c} + (1 - \rho_j)\underset{\sim}{c};\ j = 1,2, \ldots., n,$
$b'_i(\otimes) = \beta_i\tilde{b} + (1 - \beta_i)\underset{\sim}{b};\ i = 1,2, \ldots, m,$
$a'_{ij}(\otimes) = \delta_{ij}\tilde{a}_{ij} + (1 - \delta_{ij})\underset{\sim}{a}_{ij};\ i = 1,2, \ldots, m;\ \ j = 1,2, \ldots, n.$

where δ_{ij}, β_j *and* ρ_j (i =1, …, m and j = 1, …, n) lies in the closed interval [0,1].

Then,

$$
\begin{aligned}
&MaxS = C'(\otimes)X \\
&s.t. \\
&A'(\otimes)X \leq b'(\otimes), X \geq 0. \qquad (2)
\end{aligned}
$$

is called a positioned programming of grey linear programming where δ_{ij}, β_j *and* ρ_j (i = 1,2…., m and j = 1, 2, …., n) are positioned coefficient of consumption vector, constraint vector for resource and off price vector, respectively.

Definition 2. When $\rho = \beta = 1, \delta = 0$ then programming model is called ideal model. For these values we get lowest values for the model.

Definition 3. When $\rho = \beta = 0, \delta = 1$ then programming model is called critic model. For these values we get highest values of the model.

2.2 Grey BCM method – The Proposed method

In this section all the steps of GBCM methods has been described in detail as follows:

Step 1. Certify the criteria set for decision making. This step includes specification of desired criteria $(C_1, C_2, \ldots\ldots, C_n)$ for any real-world decision-making problem.

Step 2. Certify the base criterion. In this step decision maker chooses one criteria from the set of criteria as a base-criterion.

Step 3. Determine the pairwise comparison of base criterion with all other criteria. In this step relative importance of the base criterion over all the criteria's is done by using grey linguistic terms in Table 1.

Table 1. Pairwise comparison using grey number

Linguistic variable	Grey Number
Equally Important (EI)	[1,1]
Weakly Important (WI)	[2,3]
Fairly Important (FI)	[4,5]
Very Important (VI)	[6,7]
Absolutely Important (AI)	[8,9]

Step 4. Calculate the optimal weights of the criteria $(w_1, w_2, \ldots\ldots, w_n)$. The optimal weight for each $\frac{w_b}{w_j}$ will be equal to a_{Bj} for all values of j. The problem can be expressed as follows:

$$Min\,max\left|\frac{w_B}{w_j} - a_{Bj}\right|$$
$$s.t.$$
$$\begin{cases}\sum_j w_j = 1\\ w_j \geq 0, \forall j\end{cases} \quad (3)$$

We can rewrite this equation as:

$$Min\xi$$
$$s.t.$$
$$\left|\frac{w_B}{w_j} - a_{Bj}\right| \leq \xi, \forall j$$
$$\sum_j w_j = 1 \quad (4)$$
$$w_j \geq 0, \forall j$$

To convert non-linear model into linear model McCormick method has been used for linearization. Following steps has been involved:

If $\phi_1 = x_1 x_2$ and $x_1 \in [x_1^L, x_1^U], x_2 \in [x_2^U, x_2^U]$

For converting non-linear model into linear, following assumption are considered.

$\phi_1 = w_j \xi\,,\quad \mathrm{w}_j \in [0,1]$

Where, $\xi =$ CI × CR. The value of CR belongs to [0,1]. Inconsistency ratio should be less than equal to specified value A. so we can write,

$\xi = [0, \mathrm{CI} \times A],\ \mathrm{CI} \geq 0,\ 0 \leq A \leq 1.$

Then adding the given constraints to our model, it is converted into linear model:

$$\phi_1 \geq x_1^L x_2 + x_2^L x_1 - x_1^L x_2^L,$$
$$\phi_1 \geq x_1^U x_2 + x_2^U x_1 - x_1^U x_2^U,$$
$$\phi_1 \leq x_1^L x_2 + x_2^U x_1 - x_1^L x_2^U,$$
$$\phi_1 \leq x_1^U x_2 + x_2^L x_1 - x_1^U x_2^L.$$

and we get the final grey linear model as:

$$\begin{aligned} & \min \xi \\ & s.t. \\ & \otimes w_B - \otimes w_j \otimes a_{Bj} \leq \otimes \phi_1 \ \forall \ \text{j}, \\ & - \otimes w_B + \otimes w_j \otimes a_{Bj} \leq \otimes \phi_1 \ \forall \ \text{j}, \\ & \otimes \phi_1 \geq 0, \\ & \otimes \phi_1 \geq \otimes \xi + CI.A \otimes w_j - CI.A, \\ & \otimes \phi_1 \leq CI.A \otimes w_j, \\ & \otimes \phi_1 \leq \xi, \\ & \widetilde{w}_j - \underset{\sim}{w}_j \geq \varepsilon, \forall \ \text{j}, \\ & \sum_j \otimes w_j = [0.8,1.2], \\ & \otimes w_j \geq 0, \forall \ \text{j}, 0 \leq A \leq 1, \text{CI} \geq 0. \end{aligned} \tag{5}$$

To obtain the grey optimal weights, grey linear model can be solved by using positioned programming method.

Step 5. To aggregate the weight of each criteria we used grey geometric mean.

$$\otimes w_j = (w_{1j}, w_{2j}, \ldots, w_{kj})^{\frac{1}{k}}$$

Step 6. Now the calculated weights are normalized by using following equation.

$$\otimes w_j^* = \left(\frac{w_1}{\frac{1}{2}[\sum_{j=1}^n w_1 + \sum_{j=1}^n w_2]}, \frac{w_2}{\frac{1}{2}[\sum_{j=1}^n w_1 + \sum_{j=1}^n w_2]} \right) \tag{6}$$

Step 7. To compare the grey weights grey possibility degree is used.

$$\text{GP}_{ij} = \begin{bmatrix} P(\otimes A \leq \otimes A) & P(\otimes B \leq \otimes A) & \cdots & P(\otimes N \leq \otimes A) \\ P(\otimes A \leq \otimes B) & P(\otimes B \leq \otimes B) & \cdots & P(\otimes N \leq \otimes A) \\ \vdots & \vdots & \ddots & \vdots \\ P(\otimes N \leq \otimes A) & P(\otimes B \leq \otimes A) & \cdots & P(\otimes N \leq \otimes N) \end{bmatrix} \tag{7}$$

We can further write this matrix as:

$$P_{ij} = \begin{bmatrix} P_{AA} & P_{BA} & \cdots & P_{NA} \\ P_{AB} & P_{BB} & \cdots & P_{NB} \\ \vdots & \vdots & \ddots & \vdots \\ P_{AN} & P_{BN} & \cdots & P_{NN} \end{bmatrix} \tag{8}$$

where, $P_{ij} = \begin{cases} 1 & \text{P}(i \leq j) > 0.5 \quad \text{i, j = A, ..., N} \\ 0 & \text{P}(i \leq j) \leq 0.5 \quad \text{i, j = A, ..., N} \end{cases}$, then the horizontal components of the P_{ij} has been added and scores of the different criteria has been obtained. Based on these scores, the criteria has been prioritized.

Step 8. In this step, consistency ratio has been find out using following formula.

$$Consistency \text{ Ratio} = \frac{\xi}{Consistency \text{ Index}} \tag{9}$$

To determine consistency index, the following equation has been used (Rezaei, 2015).

$$\xi^2 - (1 + 2a_j)\xi + (a_j^2 - a_j) = 0, \tag{10}$$

where ξ varies from 1/9 to 9. Taking maximum of all the minimum values of ξ, we will get consistency index.

2.2.1 Numerical example

In this section, we implemented the GBCM to solve two MCDM problems. first problem is transportation mode problem (for one expert) and second problem is car selection (for three expert).

Example 1: A company needs to select an optimal transportation mode too deliver the products to a market. We solved transportation mode problem using proposed method in this paper.

Step 1. Three criteria Load Flexibility, Accessibility and Cost are selected for this company.

Step 2. Cost is chosen as the base-criterion.

Step 3. Now, pairwise comparisons are done to determine the relative importance of base-criterion to the other criteria (Table 2).

Table 2. Pairwise comparison for example 1

Base-criterion	Load flexibility	Accessibility
cost	[7, 8]	[1, 2]

Step 4. Grey linear model for this problem is formed as follows:

$$
\begin{aligned}
& min\otimes \xi \\
& s.t. \\
& \otimes w_1 - [7,8] \otimes w_2 \leq \otimes \phi_2, \\
& - \otimes w_1 + [7,8] \otimes w_2 \leq \otimes \phi_2, \\
& \otimes \phi_2 \geq 0, \\
& \otimes \phi_2 \geq \otimes \xi + 5.228 * 1 * \otimes w_2 - 5.228 * 1, \\
& \otimes \phi_2 \leq 5.228 * 1 * \otimes w_2, \\
& \otimes \phi_2 \leq \otimes \xi, \\
& \otimes w_1 - [1,2] \otimes w_3 \leq \otimes \phi_3, \\
& - \otimes w_1 + [1,2] \otimes w_3 \leq \xi * \otimes w_3, \\
& \otimes \phi_3 \geq r0, \\
& \otimes \phi_3 \geq \otimes \xi + 5.228 * 1 * \otimes w_3 - 5.228 * 1, \\
& \otimes \phi_3 \leq 5.228 * 1 * \otimes w_3, \\
& \otimes \phi_3 \leq \xi, \\
& \otimes w_1 + \otimes w_2 + \otimes w_3 = [0.8, 1.2] \\
& \otimes w_1 \geq 0, \otimes w_2 \geq 0, \otimes w_3 \geq 0.
\end{aligned} \tag{11}
$$

After solving this grey model using positioned programming technique, the weight values of the criteria are obtained. Table 3 demostrate the weight values.

Table 3. Weights for example 1

weights	Lower	Upper
$\otimes w_1$	0.4923076925	0.5600000006
$\otimes w_2$	0.0615384614	0.0800000000
$\otimes w_3$	0.2461538459	0.5599999991
$\otimes \xi$	0.0	0.0

Step 5. Further Table 4 represents the calculated normalized weights.

Table 4. Normalized weights for example 1

Weight	Lower	Upper
$\otimes w_1^*$	0.4923076925	0.5600000006
$\otimes w_2^*$	0.0615384614	0.0800000000
$\otimes w_3^*$	0.2461538459	0.5599999991
$\otimes \xi$	0.0	0.0

Step 6. This step involves formation of grey possibility degree and prioritization of the criteria. Grey possibility degree is formed as:

$$\mathrm{GP}_{ij}=\begin{bmatrix} 0.5 & 1 & 0.82 \\ 0 & 0.5 & 0 \\ 0.17 & 1 & 0.5 \end{bmatrix} \tag{12}$$

$$P_{ij}=\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{matrix} 2 \\ 0 \\ 1 \end{matrix} \tag{13}$$

Based on the sum values of horizontal component in P_{ij}, criteria are prioritized as in following order Cost > Accessibility > Load Flexibility.

Result for Transportation Mode Problem indicates that the cost criteria have major effect than accessibility and accessibility have greater effect than load flexibility. From the Table 4 we can see that value of ξ become zero which leads to the zero-consistency ratio.

Example 2: Here we are solving an another MCDM problem about selection of car based on some important criteria. Three experts are taken for multi criteria group decision making. We can determine many criteria for car selection. In the current example, five criteria are under consideration as price, quality, comfort, safety and style.

Step 1. Set of criteria determined by experts is (price, quality, comfort, safety, style).

Step 2. Each expert chooses any one criteria among the different criteria as base criterion. Here all three experts selected price as base-criterion.

Step 3. Now, using grey linguistic variables pairwise comparison (Table 5) of base-criterion to other criteria is done.

Table 5. Pairwise comparison for example 2

Experts	Base-Criterion	Quality	Comfort	Safety	Style
1	Price	[2, 3]	[6, 7]	[6, 7]	[8, 9]
2	Price	[4, 5]	[7, 8]	[8, 9]	[6, 7]
3	Price	[1/3,1/2]	[8, 9]	[6, 7]	[7, 8]

Step 4. Grey model for the first decision maker is formed as:

$min\otimes \xi$

$s.t.$

$\otimes w_1 - [2,3] \otimes w_2 \leq\otimes \emptyset_2,$

$-\otimes w_1 + [2,3] \otimes w_2 \leq\otimes \emptyset_2,$

$\otimes \emptyset_2 \geq 0,$

$\otimes \emptyset_2 \geq\otimes \xi + CI.A \otimes w_2 - CI.A,$

$\otimes \emptyset_2 \leq CI.A \otimes w_2,$

$\otimes \emptyset_2 \leq \xi,$

$\otimes w_1 - [6,7] \otimes w_3 \leq\otimes \emptyset_3,$

$-\otimes w_1 + [6,7] \otimes w_3 \leq\otimes \emptyset_3,$

$\otimes \emptyset_3 \geq 0,$

$\otimes \emptyset_3 \geq\otimes \xi + CI.A \otimes w_3 - CI.A,$

$$\begin{aligned}
&\otimes \emptyset_3 \leq CI.A \otimes w_3,\\
&\otimes \emptyset_3 \leq \xi,\\
&\otimes w_1 - [6,7] \otimes w_4 \leq \otimes \emptyset_4,\\
&- \otimes w_1 + [6,7] \otimes w_4 \leq \otimes \emptyset_4,\\
&\otimes \emptyset_4 \geq 0,\\
&\otimes \emptyset_4 \geq \otimes \xi + CI.A \otimes w_4 - CI.A,\\
&\otimes \emptyset_4 \leq CI.A \otimes w_4,\\
&\otimes \emptyset_4 \leq \xi,\\
&\otimes w_1 - [8,9] \otimes w_5 \leq \otimes \emptyset_5,\\
&- \otimes w_1 + [8,9] \otimes w_5 \leq \otimes \emptyset_5,\\
&\otimes \emptyset_5 \geq 0,\\
&\otimes \emptyset_5 \geq \otimes \xi + CI.A \otimes w_5 - CI.A,\\
&\otimes \emptyset_5 \leq CI.A \otimes w_5,\\
&\otimes \emptyset_5 \leq \xi,\\
&\otimes w_1 + \otimes w_2 + \otimes w_3 + \otimes w_4 + \otimes w_5 = [0.8,1.2]\\
&\otimes w_1 \geq 0, \otimes w_2 \geq 0, \otimes w_3 \geq 0, \otimes w_4 \geq 0, \otimes w_5 \geq 0
\end{aligned} \tag{14}$$

After solving this grey model using positioned programming method, the weights values of the criteria for the first decision maker are obtained (Table 6). Similarly, weight values can be obtained for the second and third decision maker (Table 7 and Table 8) as well.

Table 6. Weights for expert 1 in example 2

Criteria	Lower	Upper	Grey Weights
price	0.462385321	0.612765958	[0.462,0.612]
quality	0.154128440	0.306382979	[0.154,0.306]
comfort	0.066055046	0.102127659	[0.066,0.102]
safety	0.066055046	0.102127659	[0.066,0.102]
style	0.051376147	0.076595745	[0.051,0.076]
$\otimes \xi$	0.0	0.0	[0,0]

Table 7. Weights for expert 2 in example 2

Criteria	Lower	Upper	Grey Weights
price	0.506659965	0.712367492	[0.506,0.712]
quality	0.101331993	0.178091872	[0.101,0.178]
comfort	0.063332495	0.101766784	[0.063,0.101]
safety	0.056295552	0.089045936	[0.056,0.089]
style	0.072379995	0.118727915	[0.072,0.118]
$\otimes \xi$	0.0	0.0	[0,0]

Table 8. Weights for expert 3 in example 2

Criteria	Lower	Upper	Grey Weights
price	0.236758661	0.268767422	[0.236,0.268]
quality	0.473517322	0.814446733	[0.473,0.814]
comfort	0.026306518	0.033595928	[0.026,0.033]
safety	0.033822666	0.044794570	[0.033,0.044]
style	0.029594833	0.038395346	[0.029,0.038]
$\otimes \xi$	0.0	0.0	[0,0]

Step 5. Using grey geometric mean, aggregated weight are obtained as follows (Table 9).

Table 9. Aggregated weights for example 2

Weight	Lower	Upper
$\otimes w_1$	0.381366172	0.48954406
$\otimes w_2$	0.194829663	0.35420700
$\otimes w_3$	0.047921675	0.07041719
$\otimes w_4$	0.050102874	0.07413005
$\otimes w_5$	0.047921675	0.07041719

Step 6. The normalized weights are obtained as follows (Table 10).

Table 10. Normalized weights for example 2

Weight	Lower	Upper
$\otimes w_1^*$	0.428294979	0.549784635
$\otimes w_2^*$	0.218804321	0.397793750
$\otimes w_3^*$	0.053818650	0.079082339
$\otimes w_4^*$	0.056268256	0.083252082
$\otimes w_5^*$	0.053818650	0.079082339

Step 7. In this step, the criteria are propritized using grey possibility degree.

$$\mathrm{GP}_{ij} = \begin{bmatrix} 0.5 & 1 & 1 & 1 & 1 \\ 0 & 0.5 & 1 & 1 & 1 \\ 0 & 0 & 0.5 & 0.47 & 0.5 \\ 0 & 0 & 0.59 & 0.5 & 0.59 \\ 0 & 0 & 0.5 & 0 & 0.5 \end{bmatrix} \tag{15}$$

$$P_{ij} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 4 \\ 3 \\ 0 \\ 2 \\ 0 \end{matrix} \tag{16}$$

Based on the sum values of the horizontal component of the P_{ij}, criteria can prioritize as Price > Quality > Safety > Comfort > Style.

Above results show that for selection of a car price is considered as most preferable criteria. Quality of a car is second most important criteria. Further, safety is concluded as third important criteria whereas comfort and style have the same weights.

2.3 Comparative Analysis

In this section, results of numerical example by the proposed method GBCM compared with the GBWM (Mahmoudi et al. 2020). Results of GBCM are obtained by using on position programming method based on ideal and critic model. Table 11, Table 12, and Table 13 display the comparisons between proposed method and the GBWM (Mahmoudi et al. 2020).

Table 11. Grey weights and consistency ratio by the GBWM and GBCM for expert 1

	GBWM (Mahmoudi et.al, 2020) CR = [0, 0.0519]		GBCM (Proposed approach) CR = [0,0]	
Weights	Grey weights	Rank	Grey weights	Rank
$\otimes w_1$	[0.2647, 0.4000]	1	[0.462,0.612]	1
$\otimes w_2$	[0.2444, 0.3733]	2	[0.154,0.306]	2
$\otimes w_3$	[0.1047, 0.1600]	3	[0.066,0.102]	3
$\otimes w_4$	[0.1047, 0.1600]	3	[0.066,0.102]	3
$\otimes w_5$	[0.0815, 0.1067]	4	[0.051,0.076]	4

Table 12. Grey weights and consistency ratio by the GBWM and GBCM for expert 2

	GBWM (Mahmoudi et.al, 2020) CR = [0, 0.0561]		GBCM (Proposed approach) CR = [0,0]	
Weights	Grey weights	Rank	Grey weights	Rank
$\otimes w_1$	[0.3335, 0.4570]	1	[0.506,0.712]	1
$\otimes w_2$	[0.1774, 0.3185]	2	[0.101,0.178]	2
$\otimes w_3$	[0.0985, 0.1365]	4	[0.063,0.101]	4
$\otimes w_4$	[0.0639, 0.0969]	5	[0.056,0.089]	5
$\otimes w_5$	[0.1267, 0.1911]	3	[0.072,0.118]	3

Table 13. Grey weights and consistency ratio by the GBWM and GBCM for expert 3

	GBWM (Mahmoudi et.al, 2020) CR = [0, 0.0535]		GBCM (Proposed approach) CR = [0,0]	
Weights	Grey weights	Rank	Grey weights	Rank
$\otimes w_1$	[0.2517, 0.3107]	2	[0.236,0.268]	2
$\otimes w_2$	[0.2727, 0.4661]	1	[0.473,0.814]	1
$\otimes w_3$	[0.0839, 0.1036]	5	[0.026,0.033]	5
$\otimes w_4$	[0.1864, 0.1079]	3	[0.033,0.044]	3
$\otimes w_5$	[0.1332, 0.0839]	4	[0.029,0.038]	4

Results of comparison are as follows:

- Ranking of criteria using GBCM (proposed method) is same as by using GBWM (Mahmoudi et al. 2020) with different input values.
- The results obtained by using GBCM (proposed method) is more reliable than the one calculated by GBWM. Smaller consistency ratio is the main reason behind this reliability.
- In the proposed method, we do less pairwise comparison as compared to GBWM method which consists best to others as well as others to worst pair wise comparison.

3. A case study

OTT (Over the Top) are the media platforms that make possible to deliver video, TV shows and live feeds on the internet. In India, there are various OTT platforms including Amazon Prime, Hotstar Disney, Netflix, Voot, ZEE5, Sony LIV and many others. Now a days this type of technology is largely affecting the media and entertainment industry. One can prefer any OTT service based on its content, accessibility and different other factors. In the current study, we investigate the fastest growing OTT platform in India based on some important criteria. The weights of the criteria are determined by the proposed method and ranking of the OTT platform is done by using TOPSIS method.

3.1 Listing the alternatives and criteria

There are many factors which comes in mind of users for choosing any OTT platform. We have taken 4 criteria that majorly effects the popularity of any OTT platform. The considered criteria are described as follows:

1. Cost-efficient (C_1): It describes about monthly or yearly price of any OTT platform. Usually, user or decision maker chooses that platform which have less monthly or annually price with good content quality. Range of cost-efficient differs according to the platform and the content provided by it.
2. Content Language (C_2): An OTT service contains many contents as movies, reality shows, TV serial, web series, news, sports and many others. There are many languages in which these contents are available. So, the availability of more content languages may increase the use of any OTT platform.
3. Monthly Active Users (C_3): This factor indicates the monthly subscribers of any OTT platform. In our daily life we use any product or electronic equipment and many other things by reading reviews of other users after using them. Obviously, that product is preferred which have more and positive reviews. Likewise, there is a great role of users or followers of any OTT platform behind its popularity.
4. Rating (C_4): This includes giving rating stars to any platform out of 5-star scale. That platform which has greater number of stars taken as most popular platform.

In this study, cost-efficient (C_1) is taken as non-beneficial attribute while all remaining attributes content language (C_2), monthly active users (C_3), rating (C_4) are taken as beneficial attributes. The following 5 well known Over the Top platforms have taken as alternative to analyse their performance based on above mentioned attributes or criteria.

1. Disney Hotstar (A_1)
2. ZEE5 (A_2)
3. Netflix (A_3)
4. Amazon Prime Video (A_4)
5. Sony LIV (A_5)

3.2 Determination of the weights of criteria by Grey Base-Criterion Method

The data of OTT platforms in India is collected from https://dailyweblife.com/best-ott-apps-india-2020/. The decision matrix d_{ij} is given in the Table 14.

Table 14. Decision matrix

Alternatives	C_1	C_2	C_3	C_4
Disney Hotstar	1499	8	300000000	4
ZEE5	999	12	76400000	3
Netflix	999	10	13400000	4.3
Amazon Prime Video	799	10	15800000	4
Sony LIV	499	4	65000000	3

The cost-efficient (C_1) has choosen as base criterion. Further, the pairwise comparison matrix is done as shown in Table 15.

Table 15. Base-comparison matrix

Base-criterion	Content Language	Monthly Active Users	Rating
Cost-efficient	[1,2]	[2,3]	[1/9,1/8]

Grey linear model for this pairwise comparison is given as follows:

$$
\begin{aligned}
& Min \otimes \xi \\
& s.t. \\
& |\otimes w_1 - [1,2] \otimes w_2| \leq \xi * \otimes w_2, \\
& \otimes w_2 * \otimes \xi \geq 0, \\
& \otimes w_2 * \otimes \xi \geq \otimes \xi + 5.228 * 1 * \otimes w_2 - 5.228 * 1, \\
& \otimes w_2 * \otimes \xi \leq 5.228 * 1 * \otimes w_2, \\
& \otimes w_2 \leq \otimes \xi, \\
& |\otimes w_1 - [2,3] \otimes w_3| \leq \xi * \otimes w_3, \\
& \otimes w_3 * \otimes \xi \geq 0, \\
& \otimes w_3 * \otimes \xi \geq \otimes \xi + 5.228 * 1 * \otimes w_3 - 5.228 * 1, \\
& \otimes w_3 * \otimes \xi \leq 5.228 * 1 * \otimes w_3, \\
& \otimes w_3 \leq \otimes \xi, \\
& |\otimes w_1 - [1/9,1/8] \otimes w_4| \leq \xi * \otimes w_4, \\
& \otimes w_4 * \otimes \xi \geq 0, \\
& \otimes w_4 * \otimes \xi \geq \otimes \xi + 5.228 * 1 * \otimes w_4 - 5.228 * 1, \\
& \otimes w_4 * \otimes \xi \leq 5.228 * 1 * \otimes w_4, \\
& \otimes w_4 \leq \otimes \xi, \\
& \otimes w_1 + \otimes w_2 + \otimes w_3 + \otimes w_4 = [0.8,1.2] \\
& \otimes w_1, \otimes w_2, \otimes w_3 \geq 0
\end{aligned}
\tag{17}
$$

After solving this model, the obtained grey weights of the attributes are given in Table 16 and Table 17 shows the calculated normalized weights

Table 16. Weights of the attributes.

weights	lower	upper
$\otimes w_1$	0.081355932994	0.10426614476
$\otimes w_2$	0.040677966559	0.10426614539
$\otimes w_3$	0.027118644401	0.05213307237
$\otimes w_4$	0.650847456050	0.93933463747
$\otimes \xi$	0.0	0.0

Table 17. Normalized weights.

Weights	Lower	upper
$\otimes w_1^*$	0.08135593299	0.10426614476
$\otimes w_2^*$	0.04067796655	0.10426614539
$\otimes w_3^*$	0.02711864440	0.05213307237
$\otimes w_4^*$	0.65084745605	0.93933463747
$\otimes \xi$	0.0	0.0

3.3 Ranking the OTT platforms by using TOPSIS method

In this case study, the well-known TOPSIS method is used for ranking the OTT platforms based on the four criteria. Normalized decision matrix and weighted normalized decision matrix are shown in Table 18 and Table 19.

Table18. Normalized decision matrix

Alternatives	C_1	C_2	C_3	C_4
A_1	0.338201455	0.388514345	0.946362069	0.483332956
A_2	0.558948135	0.582771517	0.241006874	0.362499717
A_3	0.558948135	0.485642931	0.042270839	0.519582927
A_4	0.647246806	0.485642931	0.049841736	0.483332956
A_5	0.779694814	0.194257172	0.205045115	0.362499717

Table 19. Weighted normalized decision matrix

Alternatives	C_1	C_2	C_3	C_4
A_1	0.0313888284	0.0281564334	0.0375004093	0.3842937057
A_2	0.0518765571	0.0422346500	0.0095501042	0.2882202793
A_3	0.0518765571	0.0351955417	0.0016750183	0.4131157336
A_4	0.0600716485	0.0351955417	0.0019750216	0.3842937057
A_5	0.0723642857	0.0140782167	0.0081250887	0.2882202793

After that, ideal best and ideal worst is determined for each criteria (Table 20) and then values of s_i^- and s_i^+ are calculated (Table 21). Finally, preference score is calculated by using following formula:

$$P_i = \frac{s_i^-}{s_i^+ + s_i^-} \tag{18}$$

Table 20. Ideal best and ideal worst solution for the criteria

Alternatives	C_1	C_2	C_3	C_4
A_1	0.0313888284	0.0281564334	0.0375004093	0.3842937057
A_2	0.0518765571	0.0422346500	0.0095501042	0.2882202793
A_3	0.0518765571	0.0351955417	0.0016750183	0.4131157336
A_4	0.0600716485	0.0351955417	0.0019750216	0.3842937057
A_5	0.0723642857	0.0140782167	0.0081250887	0.2882202793
Ideal best	0.0313888284	0.0422346500	0.0375004093	0.4131157336
Ideal worst	0.0723642857	0.0140782167	0.0016750183	0.2882202793

Table 21. Preference score value for each alternative

Alternatives	s_i^+	s_i^-	P_i	Rank
A_1	0.0320765565	0.1113137287	0.7762989558	2
A_2	0.1296142010	0.0357008227	0.2489765793	4
A_3	0.0418659135	0.1283143131	0.8948605756	1
A_4	0.0544519576	0.0991324549	0.6913470794	3
A_5	0.1375992617	0.0064500704	0.0449826179	5

Based on the preference score value, alternatives are ranked as follows: Amazon Prime Video > Disney Hotstar > Netflix > ZEE5 > Sony LIV. Amazon Prime Video is found as the fastest growing OTT platform in India.

4. Conclusion

The goal of this study is to introduce the notion of GBCM which provides the decision maker a significant way to deal with incomplete and poor data with zero stability ratio, less calculation and thus can be employed on a wide range of applications. BCM is an important method for MCDM as it consists zero consistency ratio and less computations. Also, it is difficult for decision maker to exactly define the belongings and non-belongings through crisp numbers. In this situation grey numbers is a good choice as they provide range or interval to define belongings and non-belongings. By this property, grey information has been introduced in BCM decision-making method. Further, we have established the extended GBCM method. After that numerical examples are illustrated. In addition, comparison analysis has been conducted between the proposed method and GBWM method (Mahmoudi et al. 2020) which demonstrates the advantage of the proposed method. At last, in order to validate the applicability of GBCM, a case study of selecting the fastest growing OTT platform in India is presented. This approach became successful as Amazon Prime Video is found as the fastest growing OTT platform in India.

There are many good research ideas for scholars to introduce different methods for solving grey linear programming in BCM. Some other methods for linearization may be use for getting more accurate solution for decision- making problem.

References

Ayough, A., Sgargh, S. B., & Khorshidvand, B. (2023). A new integrated approach based on base-criterion and utility additive methods and its application to supplier selection problem. Expert Systems with Applications, 221, 119740.

Bai, C., & Sarkis, J. (2013). A grey-based DEMATEL model for evaluating business process management critical success factors. International Journal of Production Economics, 146(1), 281-292.

Brans, J. P., & De Smet, Y. (2016). PROMETHEE methods. In Multiple criteria decision analysis (pp. 187-219). New York: Springer.

Dang, S. L. Y., Forrest, J. (2009). On positioned solution of linear programming with grey parameters. The proceedings of the 2009 IEEE International Conference on Systems, Man and Cybernetics (pp. 751–756). San Antonio: IEEE.

Datta, S., Sahu, N., & Mahapatra, S. (2013). Robot selection based on grey-MULTIMOORA approach. Grey Systems: Theory and Application, 3(2), 201-232.

Dožić, S., Lutovac, T., & Kalić, M. (2018). Fuzzy AHP approach to passenger aircraft type selection. Journal of Air Transport Management, 68, 165-175.

Guo, S., & Zhao, H. (2017). Fuzzy best-worst multi-criteria decision-making method and its applications. Knowledge-Based Systems, 121, 23-31.

Han, H., & Trimi, S. (2018). A fuzzy TOPSIS method for performance evaluation of reverse logistics in social commerce platforms. Expert Systems with Applications, 103, 133-145.

Haseli, G., Sheikh, R., & Sana, S. S. (2020). Base-criterion on multi-criteria decision-making method and its applications. International Journal of Management Science and Engineering Management, 15(2), 79-88.

Hwang, C-L., & Youn, K. 1981. Multiple Attribute Decision Making - Methods and Application: A State of the Art Survey. New York: Springer.

Julong, D. (1989). Introduction to grey system theory. The Journal of grey system, 1(1), 1-24.

Kaviani, M. A., Yazdi, A. K., Ocampo, L., & Kusi-Sarpong, S. (2020). An integrated grey-based multi-criteria decision-making approach for supplier evaluation and selection in the oil and gas industry. Kybernetes, 49(2), 406-441.

Keršuliene, V., Zavadskas, E. K., & Turskis, Z. (2010). Selection of rational dispute resolution method by applying new step-wise weight assessment ratio analysis (SWARA). Journal of business economics and management, 11(2), 243-258.

Korhonen, P., Moskowitz, H., & Wallenius, J. (1992). Multiple criteria decision support-A review. European Journal of Operational Research, 63(3), 361-375.

Kose, E., Kabak, M., & Aplak, H. (2013). Grey theory based MCDM procedure for sniper selection problem. Grey Systems: Theory and Application, 3(1), 35-45.

Kumar, P., Singh, R. K., & Kharab, K. (2017). A comparative analysis of operational performance of Cellular Mobile Telephone Service Providers in the Delhi working area using an approach of fuzzy ELECTRE. Applied Soft Computing, 59, 438-447.

Mahmoudi, A., Mi, X., Liao, H., Feylizadeh, M. R., & Turskis, Z. (2020). Grey best-worst method for multiple experts multiple criteria decision making under uncertainty. Informatica, 31(2), 331-357.

Mardani, A., Jusoh, A., & Zavadskas, E. K. (2015). Fuzzy multiple criteria decision-making techniques and applications–Two decades review from 1994 to 2014. Expert systems with Applications, 42(8), 4126-4148.

Narang, M., Joshi, M. C., & Pal, A. K. (2021). A hybrid Fuzzy COPRAS-Base-Criterion Method for Multi-criteria Decision Making. Soft Computing, 25(13), 8391-8399.

Narang, M., Joshi, M. C., & Pal, A. K. (2022a). A Hesitant Fuzzy Multiplicative Base-criterion Multi-Criteria Group Decision Making Method. Informatica, 46, 235-242.

Narang, M., Joshi, M. C., Bisht, K., & Pal, A. (2022b). Stock portfolio selection using a new decision-making approach based on the integration of fuzzy CoCoSo with Heronian mean operator. Decision Making: Applications in Management and Engineering, 5(1), 90–112.

Opricović, S. 1998. Multicriteria optimization of civil engineering systems (in Serbian). Belgrade: Faculty of Civil Engineering.

Rezaei, J. (2015). Best-worst multi-criteria decision-making method. Omega, 53, 49-57.

Saaty, T. L. (1990). How to make a decision: the analytic hierarchy process. European journal of operational research, 48(1), 9-26.

Saaty, T. L. (2004). Decision making—the analytic hierarchy and network processes (AHP/ANP). Journal of systems science and systems engineering, 13(1), 1-35.

Zadeh, A. L., 1996. Fuzzy sets. In Fuzzy sets, fuzzy logic and fuzzy systems. Advances in Fuzzy Systems — Applications and Theory, Vol. 6 (pp. 394-432). Singapore: World Scientific.