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Double framed T-bipolar soft sets and their applications in decision making

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Abstract

Handling symmetric information that involves both positive and negative aspects, while avoiding data loss and a two-dimensional nature, has always been a problem for researchers. So far, no structure is available to address this issue. In addition to bipolarity, symmetry, and avoiding data loss, the notion of a T-bipolar soft set is more advanced. However, to address the issue of two dimensions simultaneously, we have developed the concept of double-framed T-bipolar soft sets in this manuscript. Moreover, we have defined the ideas of "AND" product, "OR" product, extended union, extended intersection, restricted union, and restricted intersection for double-framed T-bipolar soft sets. To utilize the introduced approach, we have developed an algorithm and provided an example to demonstrate its benefits and usefulness. We have utilized our work in the prioritization of advanced techniques in Digital Signal Processing (DSP). The comparative analysis of the delivered work shows the superiority of the introduced work.

Keywords: Double framed T-bipolar soft sets, AND product, OR product, Digital signal processing.

1. Introduction

The point-wise discussion about Digital signal processing (DSP) is given as:

1. DSP is essential to many different fields, including biomedical engineering, radar, audio and video compression, telecommunications, and more. It entails the manipulation of signals to improve their integrity, retrieve useful data from them, or change their format using algorithms and statistical methods. DSP is crucial to the advancement of technology because it makes data handling, storing, and transmission economical. DSP enables immediate evaluation of complicated signals by utilizing the power of digital computing, which is essential for many sophisticated systems in our modern digital age.
2. DSP is a vital field in practical math and science that deals with the interpretation, conversion, and management of digital signals. These signals are processed utilizing algorithms made to filter out, improve, shrink, or analyze the information. They may reflect everything from video and audio to electronic sensor readings and images from medical devices. DSP deals with separate signals or inputs that have been

digitally transformed so that computers may analyze them, as opposed to analog signal processing, which deals with continuous signals.

3. Apart from its significance for transmission and digital media, DSP plays an essential part in the creation of sophisticated systems in domains such as health care imaging, sonar, and radar. DSP, for instance, is used in medical diagnostics to interpret pictures from CT or MRI scans, enabling more precise diagnosis and a sharper view of the inside processes. In the same way, DSP drives modern electronic features like smart assistants, speech recognition, and real-time audio effects.
4. DSP's capacity to quickly and accurately execute intricate computations on data is one of its main features. This feature is essential in many domains, such as telecommunications, where algorithms for digital signal processing are utilized for data transmission fault detection, coding, and decoding. DSP approaches enable compression, improvement, and noise mitigation in audio and video processing, which are critical for purposes like television broadcasting, documenting, and uploading.

Benefits of DPS across the various fields:

1. DSP is capable of processing signals in real-time, which is vital for applications like audio and video communication, control systems, and radar systems
2. While initial development costs might be high, DSP systems can reduce costs in the long term by minimizing the need for expensive analog components, offering software updates, and enabling multi-functionality in a single device.
3. DSP can perform a wide range of functions, such as filtering, modulation, demodulation, spectral analysis, and more, often within the same system.
4. Modern DSP systems are optimized for low power consumption, making them suitable for portable and battery-operated devices such as smartphones, hearing aids, and wearable technology.
5. DSP is used in a variety of applications, including telecommunications, audio processing, image processing, biomedical engineering, and robotics, making it a versatile and valuable technology across industries.
6. DSP easily integrates with other digital systems and technologies, allowing for seamless data exchange, analysis, and processing in digital formats.

The graphical representation of key points provided in Table 1 is given in Figure 1.

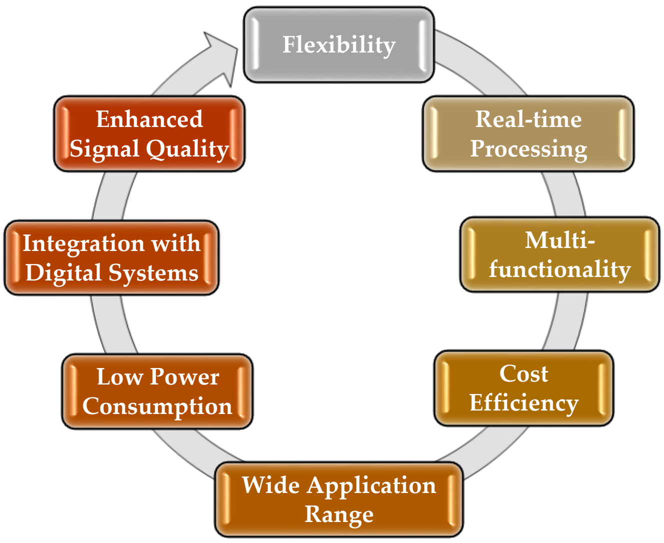

Figure 1. Graphical representation of key points of DSP

1.1 Motivation for the developed ideas

Bipolar soft sets (BSSs) are a theoretical paradigm used in decision-making and uncertainty modeling. To address situations where decision-makers have differing degrees of belief in and disbelief in an element's membership in a given set, they were introduced as an extension of classic soft sets. Bipolar soft sets are particularly useful when the data are unclear or vague. In this regard, two different sorts of efforts have been attempted. Shabir and Naz (2013) made the first attempt, and Karaaslan and Karataş (2015) made further efforts. There are certain shortcomings with the concepts presented in Shabir and Naz (2013) and Karaaslan and Karataş (2015), as noted by Mahmood (2020). The ideas proposed by Shabir and Naz (2013) and Karaaslan and Karataş (2015) are less applicable and closer to bipolarity as compared to the idea of Mahmood (2013). Moreover, we can notice that the idea of double framed soft set (SS) (Jun and Ahn, 2012) is less applicable because it cannot discuss bipolarity. To classify and prioritize the advanced techniques for DSP, a strong decision-making structure is needed that can handle the more advanced data. All the above existing notions are limited whenever decision makers try to take the information in the form of double framed T-bipolar soft sets. It can mean that there is a research gap that still exists in the literature.

DSP plays a crucial role in the efficient transmission and reception of information in modern communication systems. One area of interest within DSP is the development of advanced decision-making algorithms that enhance signal detection and classification. Double framed T-bipolar soft decision-making, which leverages the strengths of soft decision techniques to optimize performance, presents a novel approach to improve the accuracy and reliability of signal detection in the presence of noise and interference. By integrating double framed T-bipolar soft decision strategies with DSP frameworks, this research aims to investigate their effectiveness in various communication scenarios, such as wireless networks and digital video broadcasting. The exploration of this synergy could lead to significant improvements in bit error rates and overall system performance, paving the way for more robust communication solutions in increasingly complex environments. Hence, keeping in view the limitations of the existing notion Shabir and Naz (2013), Karaaslan and Karataş (2015), and Mahmood (2013), in this article, we have developed a more advanced structure that can discuss the drawbacks of the existing literature and provide more space to decision makers to handle the uncertain and imprecise information. In this article, we have introduced the notion of double framed T-bipolar soft sets (DFTBSS). Moreover, we have defined the ideas of “AND” product, “OR” product, extended union, extended intersection, restricted union, and restricted intersection for DFTBSSs. For the utilization of the introduced approach, we have initiated an algorithm for this purpose and provided an example to show the benefits and usefulness of the developed approach. The comparative analysis of the delivered work shows the superiority of the introduced work.

1.2 Arrangement of study work

The arrangement of the article is as follows: Section 1 is about the introduction of the proposed application. Section 2 discusses the literature review about the DSP as well as SS theory and their applications. In section 3, we have discussed some preliminary notions that can help to discuss further notions. Section 4 is about the fundamentals of double framed T-bipolar soft sets and their fundamental properties. In Section 5 we have developed an algorithm for the application of the proposed work. Section 6 discusses the comparative analysis of the introduced work to show the superiority of the developed approach. Section 7 is about the concluding remarks.

2. Literature review

This section is devoted to discussing the literature review about the application of the proposed work. Also, we have discussed the fundamentals of soft set structures and their application provided by different researchers. The section-wise study is given by

2.1 Literature review of the application framework

In Table 1, we have discussed some key features of DSP.

Table 1. Some Key Features of DSP

Title of features	Explanation
Historical development	The origins of DSP can be traced back to the 1960s with the advent of digital computers. Initially, it was primarily used in military and space applications, but as technology advanced, DSP became prevalent in consumer electronics, telecommunications, and multimedia systems.
Importance and Relevance in Modern Technology	Today, DSP is ubiquitous in modern technology, ranging from simple applications like noise reduction in audio devices to complex tasks like real-time signal analysis in telecommunications and medical imaging. Its relevance is due to the increasing need for efficient, reliable, and high-quality signal processing in various fields.
Fundamental idea is DSP	1. DSP operates on discrete-time signals, where the continuous analog signal is sampled at discrete intervals. Understanding discrete-time systems and their behavior is fundamental to designing effective DSP algorithms. 2. The Sampling Theorem, introduced by Claude Shannon, is a cornerstone of DSP. It says that if the rate of sampling is more than two times the highest frequency of the signal, the ongoing signal can be entirely captured by its portions and rebuilt
Filter Design	Filters are fundamental DSP elements that are employed to gather significant data or eliminate undesired elements from the signal, such as noise. Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters are the two primary varieties. For signal analysis to be efficient, these filters' execution and architecture are essential.

2.1.1 Challenges in DSP

Digital Signal Processing (DSP) involves the manipulation of digital signals through algorithms and various computational techniques. Despite its wide applications in areas such as telecommunications, audio and speech processing, radar, and biomedical engineering, DSP faces several challenges. Here are some of the key challenges given in Table 2. Moreover, Figure 2 describes the graphical representation of some key challenges.

Table 2. Key challenges in DSP

Title of challenges	Explanation
Computational Complexity	1. DSP algorithms often require intensive computations, particularly in real-time applications. This can strain processing power and resources, especially in systems with limited hardware capabilities. 2. Achieving real-time processing with minimal latency is crucial in applications like audio processing and communications, but the complexity of DSP algorithms can lead to delays.
Power Consumption	1. The need for high computational power often translates into high energy consumption, which is a significant concern in battery-powered devices such as mobile phones and IoT devices. 2. Balancing power efficiency with performance requires careful algorithm design and hardware optimization.
Quantization and Finite Precision Effects	Converting analog signals to digital involves quantization, which introduces noise and can degrade signal quality. The limited precision of digital representations can cause rounding errors and limit the dynamic range of the system, impacting the accuracy of DSP operations
Hardware Constraints	Embedded systems and other specialized hardware may have constraints on memory, processing power, and I/O bandwidth, which can limit the implementation of sophisticated DSP algorithms. Integrating DSP algorithms with hardware, especially in mixed-signal environments, poses significant challenges, such as dealing with interference and synchronization issues.
Data Handling	Processing large datasets, such as those found in big data analytics or multimedia applications, requires efficient data handling and storage solutions. Ensuring that data is processed in real time without delays or data loss is critical in many DSP applications, particularly in communications and control systems

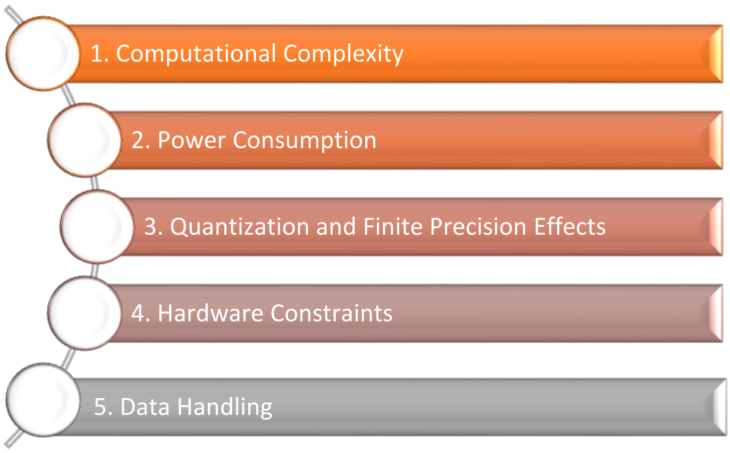

Figure 2. Graphical representation of some key challenges

Many researchers have developed their theories about DSP and found some valuable results. Hegde and Shanbhag (2001) developed the theory of soft DSP. To match the critical path delay to the throughput, they have devised a structure for low-energy DSP, in which the supply voltage is scaled beyond the critical voltage. In their research, Durand and Pibarot (1995) have offered a thorough analysis of the most current advancements in cardiac auscultation technology and the processing of signals. Cappello and Steiglitz's (1984) primary goal of the theory was to demonstrate how asymptotic complexity theory might be applied to the field of DSP. Constantinides et al. (2003) developed techniques for solving the word length allocation and optimization problem for linear DSP systems that are implemented as specialized parallel processing units. Two methods are suggested: a heuristic method and one that ensures an ideal set of word lengths for every internal variable. Tessier and Burleson (2001) provided an overview of the last fifteen years' worth of commercial and scholarly research on reconfigurable computing for DSP systems. This work is compared to other DSP implementation media that are currently accessible. Vaidyanathan and Doganata (1989) described in detail the discrete-time lossless structures, their characteristics, and their applicability to DSP. A few algebraic features of lossless systems are examined, and the fundamental idea of losslessness is presented. Before moving on to linear operators between two Hilbert spaces, Biglieri and Yao (1989) first go over some fundamental aspects of singular value decomposition for matrices. Sahambi et al. (1997) discussed the drawbacks of other techniques for detecting QRS (The combination of Q-waves, R-waves, and S-waves) and the onsets and offsets of P- and T-waves, which outlines a real-time system that makes use of wavelet transforms. A brief discussion of wavelet transformation is followed by the presentation of the system's hardware, software, detection techniques, and experimental findings. In coherent optical fiber transmission systems, Cartledge et al. (2017) examined DSP methods that adjust for, alleviate, and make use of fiber nonlinearities. Van Straten and Bailes (2011) analyzed the phase-coherent dispersion removal techniques, justified key design choices, and provided a thorough explanation of their functionality as well as performance results on a few modern microprocessor architectures.

2.2 Literature review about soft set structures

A soft set (SS) is an algebraic framework that builds upon the concept of a classical (crisp) set to account for any uncertainty or ambiguity in data. In 1999, Molodtsov (1999) developed SSs as a technique to deal with ambiguous data in analysis and decisions. A popular mathematical framework for managing hazy and unclear data that cannot be handled by other traditional methods is Molodtsov SS theory. By considering the parameterization process, the creation of an SS turned out to be a very intriguing method for tackling the problem. Recently, there has been a greater interest in applying the SS theory as a way to resolve unclear data. SSs have proven beneficial in numerous fields, like multi-attribute decision-making (MADM) problems (Zahedi Khameneh and Kılıçman, 2019), stock management (Taş et al., 2017), dimensionality reduction (Herawan et al, 2010), ideals in d-algebra (Jun et al., 2009), and computational biology (Santos-Buitrago et al., 2019). Çağman and Enginoğlu (2010) improved some new results by redefining the operations of Molodtsov SSs to make them more functional. Additionally, they have defined the uni-int decision function and products of SSs. Next, they have created a uni-int decision-making process utilizing these new definitions, which chooses a set of ideal elements from the available options. Ali et al. (2009) presented some new ideas, including the extended intersection of two SSs, the restricted difference, the restricted union, and the restricted intersection. Additionally, they have refined the concept of an SS complement and demonstrate that, in light of these new definitions, certain of De Morgan's rules still apply to SS theory. Jun et al. (2008) introduced the concepts of commutative soft ideals and commutative idealistic soft BCK-algebras and examined their fundamental features. They have extended the idea of SSs by Molodtsov to commutative ideals of BCK-algebras. A potent mathematical tool for handling uncertainties beyond the typical SS theory formulation is the notion of N-soft sets (N-SS). In their study, Akram et al. (2021) have justified the practical computation of parameter reduction and expanded its concept to N-SS theory. To achieve this, they have defined relevant theoretical terms and looked at some of their basic characteristics.

3. Preliminaries

In our next discussion, we utilize Ụ as a universal set and $X \subseteq \text{Ụ}$ and $Y = \text{Ụ} - X$ with $\text{ス} \subseteq E$ the set of parameters.

Definition 1 (Molodtsov, 1999): Let Ụ be the universal set, and E be the set of parameters with $\text{ス} \subseteq E$. Then the structure of the form $(\dot{f}, \text{ス})$ is said to be SS over Ụ, where $\dot{f}: \text{ス} \to P(\text{Ụ})$.

Definition 2 (Ali et al, 2009)**:** Let Ụ be a universal set and E be the set of parameters and $\text{ス}_1, \text{ス}_2 \subset E$. Also, assume that $(\dot{f}_1, \text{ス}_1)$ and $(\dot{f}_2, \text{ス}_2)$ denote the SSs over Ụ, then the extended union of these two SSs is $(\dot{f}_1, \text{ス}_1) \cup_{\text{ẹ}xt.} (\dot{f}_2, \text{ス}_2) = (\dot{f}_3, \text{ス}_1 \cup \text{ス}_2)$ where

$$\dot{f}_3(\text{ẹ}) = \begin{cases} \dot{f}_1(\text{ẹ}) & ;\ if\ \text{ẹ} \in \text{ス}_1 - \text{ス}_2 \\ \dot{f}_2(\text{ẹ}) & ;\ if\ \text{ẹ} \in \text{ス}_2 - \text{ス}_1 \\ \dot{f}_1(\text{ẹ}) \cup \dot{f}_2(\text{ẹ}) & ;\ if\ \text{ẹ} \in \text{ス}_1 \cap \text{ス}_2 \end{cases} \tag{1}$$

And

$(\dot{f}_1, \text{ス}_1) \cap_{\text{ẹ}xt.} (\dot{f}_2, \text{ス}_2) = (\dot{f}_3, \text{ス}_1 \cap \text{ス}_2)$ where

$$\dot{f}_3(\text{ẹ}) = \begin{cases} \dot{f}_1(\text{ẹ}) & ;\ if\ \text{ẹ} \in \text{ス}_1 - \text{ス}_2 \\ \dot{f}_2(\text{ẹ}) & ;\ if\ \text{ẹ} \in \text{ス}_2 - \text{ス}_1 \\ \dot{f}_1(\text{ẹ}) \cap \dot{f}_2(\text{ẹ}) & ;\ if\ \text{ẹ} \in \text{ス}_1 \cap \text{ス}_2 \end{cases} \tag{2}$$

Definition 3 (Shabir and Naz, 2013): Let Ụ be the universal set, $X \subset \text{Ụ}$, and E be the set of parameters with $\text{ス} \subseteq E$. Then the structure of the form $\langle\big((\dot{f}, \breve{g}); \text{ス}\big)\rangle$ is said to be double framed SS over Ụ, where $\dot{f}: \text{ス} \to P(X), \breve{g}: \text{ス} \to P(X)$.

Definition 4 (Mahmood, 2020): Assume that Ụ represents the universal set and E is the set of parameters, and $\text{ス} \subset E$. Also, let $X \subset \text{Ụ}$ and $Y = \text{Ụ} - X$. then $(\dot{f}, \breve{g}, \text{ス})$ is said to be TBSS over Ụ, where $\dot{f}: \text{ス} \to P(X)$ and $\breve{g}: \text{ス} \to P(Y)$. So, TBSS is given by simply $(\dot{f}, \breve{g}, \text{ス}) = \{\text{ẹ}, \dot{f}(\text{ẹ}), \breve{g}(\text{ẹ}): \dot{f}(\text{ẹ}) \in P(X)\ and\ \breve{g}(\text{ẹ}) \in P(Y)\}$.

4. Double framed T-bipolar soft set

Definition 5: Let Ụ be the universal set, $X \subset \text{Ụ}$, and $Y = \text{Ụ} - X$, and E be the set of parameters with $\text{ス} \subseteq E$. Then the structure of the form $\langle\big((\dot{f}, \breve{g}); \text{ス}\big), \big((\gamma, \delta); \text{ス}\big)\rangle$ is said to be a double framed T-bipolar soft set over Ụ, where $\dot{f}: \text{ス} \to P(X), \breve{g}: \text{ス} \to P(X)$ and $\gamma: \text{ス} \to P(Y), \delta: \text{ス} \to P(Y)$.

Definition 6: Let Ụ be the universal set, $X, Y \subseteq \text{Ụ}$ and E be the set of parameters with $\text{ス} \subseteq E$. Then, the support of double framed T-bipolar soft set is given by

$$supp\ \langle\big((\dot{f}, \breve{g}); \text{ス}\big), \big((\gamma, \delta); \text{ス}\big)\rangle = \{x \in \text{ス}\ such\ that\ \dot{f}(x) \neq \emptyset \neq \breve{g}(x)\ and\ \gamma(x) \neq \emptyset \neq \delta(x)\} \tag{3}$$

Definition 7: For two double framed T-bipolar soft sets $\langle\big((\dot{f}_1, \breve{g}_1); \text{ス}\big), \big((\gamma_1, \delta_1); \text{ス}\big)\rangle$ and $\langle\big((\dot{f}_2, \breve{g}_2); B\big), \big((\gamma_2, \delta_2); B\big)\rangle$ over universal set Ụ. Then we say that $\langle\big((\dot{f}, \breve{g}); \text{ス}\big), \big((\gamma, \delta); \text{ス}\big)\rangle$ is double framed T-bipolar soft subset of $\langle\big((\dot{f}, \breve{g}); B\big), \big((\gamma, \delta); B\big)\rangle$ if

$$\text{ス} \subseteq B \tag{4}$$

$$\dot{f}_1(x) \subseteq \dot{f}_1(x), \breve{g}_1(x) \supseteq \breve{g}_2(x) \text{ and } \delta_1(x) \supseteq \dot{f}_1(x), \delta_1(x) \subseteq \delta_2(x) \tag{5}$$

for all $x \in supp\ \langle ((\dot{f},\breve{g}); \rightthreetimes), ((\gamma,\delta); \rightthreetimes)\rangle$

Definition 8: For two double framed T-bipolar soft sets $\{((\dot{f}_1,\breve{g}_1); \rightthreetimes), ((\gamma_1,\delta_1); \rightthreetimes)\}$ and $\langle ((\dot{f}_2,\breve{g}_2); B), ((\gamma_2,\delta_2); B)\rangle$ over universal set $\underset{\sim}{U}$. The idea of the "AND" product of two double framed T-bipolar soft sets is denoted and defined by

$$\langle ((\dot{f}_1,\breve{g}_1); \rightthreetimes), ((\gamma_1,\delta_1); \rightthreetimes)\rangle \wedge \langle ((\dot{f}_2,\breve{g}_2); B), ((\gamma_2,\delta_2); B)\rangle = \langle ((\dot{f}_3,\breve{g}_3); \rightthreetimes \times B), ((\gamma_3,\delta_3); \rightthreetimes \times B)\rangle \quad (6)$$

Where

$\dot{f}_3(x,y) = \dot{f}_1(x) \cap \dot{f}_2(y)$ and $\breve{g}_3(x,y) = \breve{g}_1(x) \cup \breve{g}_2(y)$ for all $(x,y) \in \rightthreetimes \times B$. Also $\gamma_3(x,y) = \gamma_1(x) \cup \gamma_2(y)$ and $\delta_3(x,y) = \delta_1(x) \cap \delta_2(y)$.

In other words

$$\begin{aligned}
&\langle ((\dot{f}_1,\breve{g}_1); \rightthreetimes), ((\gamma_1,\delta_1); \rightthreetimes)\rangle \wedge \langle ((\dot{f}_2,\breve{g}_2); B), ((\gamma_2,\delta_2); B)\rangle \\
&= \langle ((\dot{f}_3,\breve{g}_3); \rightthreetimes \times B), ((\gamma_3,\delta_3); \rightthreetimes \times B)\rangle \\
&= \left\{ \begin{matrix} (x,y), \dot{f}_3(x,y) = \dot{f}_1(x) \cap \dot{f}_2(y), \breve{g}_3(x,y) = \breve{g}_1(x) \cup \breve{g}_2(y), \\ \gamma_3(x,y) = \gamma_1(x) \cup \gamma_2(y), \delta_3(x,y) = \delta_1(x) \cap \delta_2(y) \end{matrix} \right\} \quad (7)
\end{aligned}$$

Example 1: Let $\underset{\sim}{U} = \{¢_1, ¢_2, ¢_3, ¢_4, ¢_5, ¢_6\}$**,** $X \subset \underset{\sim}{U} = \{¢_1, ¢_3, ¢_5\}$ and $Y = \underset{\sim}{U} - X = \{¢_2, ¢_4, ¢_6\}$. Assume that $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $\rightthreetimes \subset E = \{e_1, e_2, e_5\}, B \subset E = \{e_1, e_4, e_5, e_6\}$. Now $\langle ((\dot{f}_1,\breve{g}_1); \rightthreetimes), ((\gamma_1,\delta_1); \rightthreetimes)\rangle$ and $\langle ((\dot{f}_2,\breve{g}_2); B), ((\gamma_2,\delta_2); B)\rangle$ are two DFTBSS where $\dot{f}_1: \rightthreetimes \to P(X), \breve{g}_1: \rightthreetimes \to P(X)$ and $\gamma_1: \rightthreetimes \to P(Y), \delta_1: \rightthreetimes \to P(Y)$ and $\dot{f}_2: B \to P(X), \breve{g}_2: B \to P(X)$ and $\gamma_2: \rightthreetimes \to P(Y), \delta_2: \rightthreetimes \to P(Y)$ are defined by

$\dot{f}_1(e_1) = \{¢_1, ¢_3\}, \dot{f}_1(e_2) = \{¢_1, ¢_5\}, \dot{f}_1(e_5) = \{¢_1, ¢_3, ¢_5\}, \breve{g}_1(e_1) = \{¢_1\}, \breve{g}_1(e_2) = \{¢_1, ¢_5\}, \breve{g}_1(e_5) = \{¢_1, ¢_3\}$

$\gamma_1(e_1) = \{¢_2\}, \gamma_1(e_2) = \{¢_2, ¢_4\}, \gamma_1(e_5) = \{¢_2, ¢_6\}, \delta_1(e_1) = \{¢_6\}, \delta_1(e_2) = \{¢_2, ¢_6\}, \delta_1(e_5) = \{¢_4, ¢_6\}$

Now

$\dot{f}_2(e_1) = \{¢_1, ¢_3\}, \dot{f}_2(e_4) = \{¢_1, ¢_5\}, \dot{f}_2(e_5) = \{¢_1, ¢_3, ¢_5\}, \dot{f}_2(e_6) = \{¢_1, ¢_3, ¢_5\},$
$\breve{g}_2(e_1) = \{¢_1\}, \breve{g}_2(e_4) = \{¢_1, ¢_5\}, \breve{g}_2(e_5) = \{¢_1, ¢_3\}, \breve{g}_2(e_6) = \{¢_1, ¢_3\},$
$\gamma_2(e_1) = \{¢_4\}, \gamma_2(e_4) = \{¢_4, ¢_6\}, \gamma_2(e_5) = \{¢_6\}, \gamma_2(e_6) = \{¢_2, ¢_4, ¢_6\},$
$\delta_2(e_1) = \{¢_4, ¢_6\}, \delta_2(e_4) = \{¢_2, ¢_4\}, \delta_2(e_5) = \{¢_2, ¢_6\}, \delta_2(e_6) = \{¢_6\}.$

Now as $\rightthreetimes \times B = \left\{ \begin{matrix} (e_1,e_1), (e_1,e_4), (e_1,e_5), (e_1,e_6), (e_2,e_1), (e_2,e_4), (e_2,e_5), (e_2,e_6), \\ (e_5,e_1), (e_5,e_4), (e_5,e_5), (e_5,e_6) \end{matrix} \right\}$

Notice that

$\dot{f}_3(e_1,e_1) = \dot{f}_1(e_1) \cap \dot{f}_2(e_1) = \{¢_1, ¢_3\} \cap \{¢_1, ¢_3\} = \{¢_1, ¢_3\},$
$\dot{f}_3(e_1,e_4) = \dot{f}_1(e_1) \cap \dot{f}_2(e_4) = \{¢_1, ¢_3\} \cap \{¢_1, ¢_5\} = \{¢_1\},$
$\dot{f}_3(e_1,e_5) = \dot{f}_1(e_1) \cap \dot{f}_2(e_5) = \{¢_1, ¢_3\} \cap \{¢_1, ¢_3, ¢_5\} = \{¢_1, ¢_3\},$
$\dot{f}_3(e_1,e_6) = \dot{f}_1(e_1) \cap \dot{f}_2(e_6) = \{¢_1, ¢_3\} \cap \{¢_1, ¢_3, ¢_5\} = \{¢_1, ¢_3\},$
$\dot{f}_3(e_2,e_1) = \dot{f}_1(e_2) \cap \dot{f}_2(e_1) = \{¢_1, ¢_5\} \cap \{¢_1, ¢_3\} = \{¢_1\},$
$\dot{f}_3(e_2,e_4) = \dot{f}_1(e_2) \cap \dot{f}_2(e_4) = \{¢_1, ¢_5\} \cap \{¢_1, ¢_5\} = \{¢_1, ¢_5\},$
$\dot{f}_3(e_2,e_5) = \dot{f}_1(e_2) \cap \dot{f}_2(e_5) = \{¢_1, ¢_5\} \cap \{¢_1, ¢_3, ¢_5\} = \{¢_1, ¢_5\},$
$\dot{f}_3(e_2,e_6) = \dot{f}_1(e_2) \cap \dot{f}_2(e_6) = \{¢_1, ¢_5\} \cap \{¢_1, ¢_3, ¢_5\} = \{¢_1, ¢_5\},$
$\dot{f}_3(e_5,e_1) = \dot{f}_1(e_5) \cap \dot{f}_2(e_1) = \{¢_1, ¢_3, ¢_5\} \cap \{¢_1, ¢_3\} = \{¢_1, ¢_3\},$
$\dot{f}_3(e_5,e_4) = \dot{f}_1(e_5) \cap \dot{f}_2(e_4) = \{¢_1, ¢_3, ¢_5\} \cap \{¢_1, ¢_5\} = \{¢_1, ¢_5\},$
$\dot{f}_3(e_5,e_5) = \dot{f}_1(e_5) \cap \dot{f}_2(e_5) = \{¢_1, ¢_3, ¢_5\} \cap \{¢_1, ¢_3, ¢_5\} = \{¢_1, ¢_3, ¢_5\},$
$\dot{f}_3(e_5,e_6) = \dot{f}_1(e_5) \cap \dot{f}_2(e_6) = \{¢_1, ¢_3, ¢_5\} \cap \{¢_1, ¢_3, ¢_5\} = \{¢_1, ¢_3, ¢_5\}.$

Also

$\breve{g}_3(e_1,e_1) = \breve{g}_1(e_1) \cup \breve{g}_2(e_1) = \{¢_1\} \cup \{¢_1\} = \{¢_1\},$
$\breve{g}_3(e_1,e_4) = \breve{g}_1(e_1) \cup \breve{g}_2(e_4) = \{¢_1\} \cup \{¢_1, ¢_5\} = \{¢_1, ¢_5\},$

$\breve{g}_3(ȩ_1, ȩ_5) = \breve{g}_1(ȩ_1) \cup \breve{g}_2(ȩ_5) = \{¢_1\} \cup \{¢_1, ¢_3\} = \{¢_1, ¢_3\}$,
$\breve{g}_3(ȩ_1, ȩ_6) = \breve{g}_1(ȩ_1) \cup \breve{g}_2(ȩ_6) = \{¢_1\} \cup \{¢_1, ¢_3\} = \{¢_1, ¢_3\}$,
$\breve{g}_3(ȩ_2, ȩ_1) = \breve{g}_1(ȩ_2) \cup \breve{g}_2(ȩ_1) = \{¢_1, ¢_5\} \cup \{¢_1\} = \{¢_1, ¢_5\}$,
$\breve{g}_3(ȩ_2, ȩ_4) = \breve{g}_1(ȩ_2) \cup \breve{g}_2(ȩ_4) = \{¢_1, ¢_5\} \cup \{¢_1, ¢_5\} = \{¢_1, ¢_5\}$,
$\breve{g}_3(ȩ_2, ȩ_5) = \breve{g}_1(ȩ_2) \cup \breve{g}_2(ȩ_5) = \{¢_1, ¢_5\} \cup \{¢_1, ¢_3\} = \{¢_1, ¢_3, ¢_5\}$,
$\breve{g}_3(ȩ_2, ȩ_6) = \breve{g}_1(ȩ_2) \cup \breve{g}_2(ȩ_6) = \{¢_1, ¢_5\} \cup \{¢_1, ¢_3\} = \{¢_1, ¢_3, ¢_5\}$,
$\breve{g}_3(ȩ_5, ȩ_1) = \breve{g}_1(ȩ_5) \cup \breve{g}_2(ȩ_1) = \{¢_1, ¢_3\} \cup \{¢_1\} = \{¢_1, ¢_3\}$,
$\breve{g}_3(ȩ_5, ȩ_4) = \breve{g}_1(ȩ_5) \cup \breve{g}_2(ȩ_4) = \{¢_1, ¢_3\} \cup \{¢_1, ¢_5\} = \{¢_1, ¢_3, ¢_5\}$,
$\breve{g}_3(ȩ_5, ȩ_5) = \breve{g}_1(ȩ_5) \cup \breve{g}_2(ȩ_5) = \{¢_1, ¢_3\} \cup \{¢_1, ¢_3\} = \{¢_1, ¢_3\}$,
$\breve{g}_3(ȩ_5, ȩ_6) = \breve{g}_1(ȩ_5) \cup \breve{g}_2(ȩ_6) = \{¢_1, ¢_3\} \cup \{¢_1, ¢_3\} = \{¢_1, ¢_3\}$.

Moreover,

$\gamma_3(ȩ_1, ȩ_1) = \gamma_1(ȩ_1) \cup \gamma_2(ȩ_1) = \{¢_2\} \cup \{¢_4\} = \{¢_2, ¢_4\}$,
$\gamma_3(ȩ_1, ȩ_4) = \gamma_1(ȩ_1) \cup \gamma_2(ȩ_4) = \{¢_2\} \cup \{¢_4, ¢_6\} = \{¢_2, ¢_4, ¢_6\}$,
$\gamma_3(ȩ_1, ȩ_5) = \gamma_1(ȩ_1) \cup \gamma_2(ȩ_5) = \{¢_2\} \cup \{¢_6\} = \{¢_2, ¢_6\}$,
$\gamma_3(ȩ_1, ȩ_6) = \gamma_1(ȩ_1) \cup \gamma_2(ȩ_6) = \{¢_2\} \cup \{¢_2, ¢_4, ¢_6\} = \{¢_2, ¢_4, ¢_6\}$,
$\gamma_3(ȩ_2, ȩ_1) = \gamma_1(ȩ_2) \cup \gamma_2(ȩ_1) = \{¢_2, ¢_4\} \cup \{¢_4\} = \{¢_2, ¢_4\}$,
$\gamma_3(ȩ_2, ȩ_4) = \gamma_1(ȩ_2) \cup \gamma_2(ȩ_4) = \{¢_2, ¢_4\} \cup \{¢_4, ¢_6\} = \{¢_2, ¢_4, ¢_6\}$,
$\gamma_3(ȩ_2, ȩ_5) = \gamma_1(ȩ_2) \cup \gamma_2(ȩ_5) = \{¢_2, ¢_4\} \cup \{¢_6\} = \{¢_2, ¢_4, ¢_6\}$,
$\gamma_3(ȩ_2, ȩ_6) = \gamma_1(ȩ_2) \cup \gamma_2(ȩ_6) = \{¢_2, ¢_4\} \cup \{¢_2, ¢_4, ¢_6\} = \{¢_2, ¢_4, ¢_6\}$,
$\gamma_3(ȩ_5, ȩ_1) = \gamma_1(ȩ_5) \cup \gamma_2(ȩ_1) = \{¢_2, ¢_6\} \cup \{¢_4\} = \{¢_2, ¢_4, ¢_6\}$,
$\gamma_3(ȩ_5, ȩ_4) = \gamma_1(ȩ_5) \cup \gamma_2(ȩ_4) = \{¢_2, ¢_6\} \cup \{¢_4, ¢_6\} = \{¢_2, ¢_4, ¢_6\}$,
$\gamma_3(ȩ_5, ȩ_5) = \gamma_1(ȩ_5) \cup \gamma_2(ȩ_5) = \{¢_2, ¢_6\} \cup \{¢_6\} = \{¢_2, ¢_6\}$,
$\gamma_3(ȩ_5, ȩ_6) = \gamma_1(ȩ_5) \cup \gamma_2(ȩ_6) = \{¢_2, ¢_6\} \cup \{¢_2, ¢_4, ¢_6\} = \{¢_2, ¢_4, ¢_6\}$.

Also

$\delta_3(ȩ_1, ȩ_1) = \delta_1(ȩ_1) \cap \delta_2(ȩ_1) = \{¢_6\} \cap \{¢_4, ¢_6\} = \{¢_6\}$,
$\delta_3(ȩ_1, ȩ_4) = \delta_1(ȩ_1) \cap \delta_2(ȩ_4) = \{¢_6\} \cap \{¢_2, ¢_4\} = \emptyset$,
$\delta_3(ȩ_1, ȩ_5) = \delta_1(ȩ_1) \cap \delta_2(ȩ_5) = \{¢_6\} \cap \{¢_2, ¢_6\} = \{¢_6\}$,
$\delta_3(ȩ_1, ȩ_6) = \delta_1(ȩ_1) \cap \delta_2(ȩ_6) = \{¢_6\} \cap \{¢_6\} = \{¢_6\}$,
$\delta_3(ȩ_2, ȩ_1) = \delta_1(ȩ_2) \cap \delta_2(ȩ_1) = \{¢_2, ¢_6\} \cap \{¢_4, ¢_6\} = \{¢_6\}$,
$\delta_3(ȩ_2, ȩ_4) = \delta_1(ȩ_2) \cap \delta_2(ȩ_4) = \{¢_2, ¢_6\} \cap \{¢_2, ¢_4\} = \{¢_2\}$,
$\delta_3(ȩ_2, ȩ_5) = \delta_1(ȩ_2) \cap \delta_2(ȩ_5) = \{¢_2, ¢_6\} \cap \{¢_2, ¢_6\} = \{¢_2, ¢_6\}$,
$\delta_3(ȩ_2, ȩ_6) = \delta_1(ȩ_2) \cap \delta_2(ȩ_6) = \{¢_2, ¢_6\} \cap \{¢_6\} = \{¢_6\}$,
$\delta_3(ȩ_5, ȩ_1) = \delta_1(ȩ_5) \cap \delta_2(ȩ_1) = \{¢_4, ¢_6\} \cap \{¢_4, ¢_6\} = \{¢_4, ¢_6\}$,
$\delta_3(ȩ_5, ȩ_4) = \delta_1(ȩ_5) \cap \delta_2(ȩ_4) = \{¢_4, ¢_6\} \cap \{¢_2, ¢_4\} = \{¢_4\}$,
$\delta_3(ȩ_5, ȩ_5) = \delta_1(ȩ_5) \cap \delta_2(ȩ_5) = \{¢_4, ¢_6\} \cap \{¢_2, ¢_6\} = \{¢_6\}$,
$\delta_3(ȩ_5, ȩ_6) = \delta_1(ȩ_5) \cap \delta_2(ȩ_6) = \{¢_4, ¢_6\} \cap \{¢_6\} = \{¢_6\}$.

Definition 9: For two double framed T-bipolar soft sets $\left\{\left((\dot{f}_1, \breve{g}_1); ℷ\right), \left((\gamma_1, \delta_1); ℷ\right)\right\}$ and $\langle\left((\dot{f}_2, \breve{g}_2); B\right), \left((\gamma_2, \delta_2); B\right)\rangle$ over universal set $\underset{\sim}{U}$. The idea of the "OR" product of two double framed T-bipolar soft sets is denoted and defined by

$$\langle\left((\dot{f}_1, \breve{g}_1); ℷ\right), \left((\gamma_1, \delta_1); ℷ\right)\rangle \vee \langle\left((\dot{f}_2, \breve{g}_2); B\right), \left((\gamma_2, \delta_2); B\right)\rangle = \langle\left((\dot{f}_3, \breve{g}_3); ℷ \times B\right), \left((\gamma_3, \delta_3); ℷ \times B\right)\rangle \quad (8)$$

Where

$\dot{f}_3(x, y) = \dot{f}_1(x) \cup \dot{f}_2(y)$ and $\breve{g}_3(x, y) = \breve{g}_1(x) \cap \breve{g}_2(y)$ for all $(x, y) \in ℷ \times B$. Also $\gamma_3(x, y) = \gamma_1(x) \cap \gamma_2(y)$ and $\delta_3(x, y) = \delta_1(x) \cup \delta_2(y)$.

In other words

$$\langle\left((\dot{f}_1,\breve{g}_1);\ \overline{\swarrow}\right),\left((\gamma_1,\delta_1);\ \overline{\swarrow}\right)\rangle \vee \langle\left((\dot{f}_2,\breve{g}_2);B\right),\left((\gamma_2,\delta_2);B\right)\rangle$$
$$=\langle\left((\dot{f}_3,\breve{g}_3);\ \overline{\swarrow}\times B\right),\left((\gamma_3,\delta_3);\ \overline{\swarrow}\times B\right)\rangle$$
$$=\begin{Bmatrix}(x,y),\dot{f}_3(x,y)=\dot{f}_1(x)\cup\dot{f}_2(y),\breve{g}_3(x,y)=\breve{g}_1(x)\cap\breve{g}_2(y),\\ \gamma_3(x,y)=\gamma_1(x)\cap\gamma_2(y),\delta_3(x,y)=\delta_1(x)\cup\delta_2(y)\end{Bmatrix} \quad (9)$$

Example 2: Let $\underset{\smile}{U}=\{\text{¢}_1,\text{¢}_2,\text{¢}_3,\text{¢}_4\}$**,** $X\subset\underset{\smile}{U}=\{\text{¢}_1,\text{¢}_2\}$ and $Y=\underset{\smile}{U}-X=\{\text{¢}_3,\text{¢}_4\}$. Assume that $E=\{ẹ_1,ẹ_2,ẹ_3\}$ and $\overline{\swarrow}\subset E=\{ẹ_1,ẹ_2\},B\subset E=\{ẹ_2,ẹ_3\}$. Now $\langle\left((\dot{f}_1,\breve{g}_1);\ \overline{\swarrow}\right),\left((\gamma_1,\delta_1);\ \overline{\swarrow}\right)\rangle$ and $\langle\left((\dot{f}_2,\breve{g}_2);B\right),\left((\gamma_2,\delta_2);B\right)\rangle$ are two DFTBSS where $\dot{f}_1:\ \overline{\swarrow}\to P(X),\breve{g}_1:\ \overline{\swarrow}\to P(X)$ and $\gamma_1:\ \overline{\swarrow}\to P(Y),\delta_1:\ \overline{\swarrow}\to P(Y)$ and $\dot{f}_2:B\to P(X),\breve{g}_2:B\to P(X)$ and $\gamma_2:\ \overline{\swarrow}\to P(Y),\delta_2:\ \overline{\swarrow}\to P(Y)$ are defined by

$\dot{f}_1(ẹ_1)=\{\text{¢}_1\},\dot{f}_1(ẹ_2)=\{\text{¢}_1,\text{¢}_2\},\breve{g}_1(ẹ_1)=\{\text{¢}_1\},\breve{g}_1(ẹ_2)=\{\text{¢}_2\},$
$\gamma_1(ẹ_1)=\{\text{¢}_3\},\gamma_1(ẹ_2)=\{\text{¢}_3,\text{¢}_4\},\delta_1(ẹ_1)=\emptyset,\delta_1(ẹ_2)=\{\text{¢}_4\}.$

Now

$\dot{f}_2(ẹ_2)=\{\text{¢}_1,\text{¢}_2\},\dot{f}_2(ẹ_3)=\{\text{¢}_1\},$
$\breve{g}_2(ẹ_2)=\{\text{¢}_1\},\breve{g}_2(ẹ_3)=\{\text{¢}_2\},$
$\gamma_2(ẹ_2)=\{\text{¢}_4\},\gamma_2(ẹ_3)=\{\text{¢}_3\},$
$\delta_2(ẹ_2)=\{\text{¢}_3,\text{¢}_4\},\delta_2(ẹ_3)=\{\text{¢}_4\}.$

Now as $\overline{\swarrow}\times B=\{(ẹ_1,ẹ_2),(ẹ_1,ẹ_3),(ẹ_2,ẹ_2),(ẹ_2,ẹ_3)\}$

$\dot{f}_3(ẹ_1,ẹ_2)=\dot{f}_1(ẹ_1)\cup\dot{f}_2(ẹ_2)=\{\text{¢}_1\}\cup\{\text{¢}_1,\text{¢}_2\}=\{\text{¢}_1,\text{¢}_2\},$
$\dot{f}_3(ẹ_1,ẹ_3)=\dot{f}_1(ẹ_1)\cup\dot{f}_2(ẹ_3)=\{\text{¢}_1\}\cup\{\text{¢}_1\}=\{\text{¢}_1\},$
$\dot{f}_3(ẹ_2,ẹ_2)=\dot{f}_1(ẹ_2)\cup\dot{f}_2(ẹ_2)=\{\text{¢}_1,\text{¢}_2\}\cup\{\text{¢}_1,\text{¢}_2\}=\{\text{¢}_1,\text{¢}_2\},$
$\dot{f}_3(ẹ_2,ẹ_3)=\dot{f}_1(ẹ_2)\cup\dot{f}_2(ẹ_3)=\{\text{¢}_1,\text{¢}_2\}\cup\{\text{¢}_1\}=\{\text{¢}_1,\text{¢}_2\}.$

Also

$\breve{g}_3(ẹ_1,ẹ_2)=\breve{g}_1(ẹ_1)\cap\breve{g}_2(ẹ_2)=\{\text{¢}_1\}\cap\{\text{¢}_1\}=\{\text{¢}_1\},$
$\breve{g}_3(ẹ_1,ẹ_3)=\breve{g}_1(ẹ_1)\cap\breve{g}_2(ẹ_3)=\{\text{¢}_1\}\cap\{\text{¢}_2\}=\emptyset,$
$\breve{g}_3(ẹ_2,ẹ_2)=\breve{g}_1(ẹ_2)\cap\breve{g}_2(ẹ_2)=\{\text{¢}_2\}\cap\{\text{¢}_1\}=\emptyset,$
$\breve{g}_3(ẹ_2,ẹ_3)=\breve{g}_1(ẹ_2)\cap\breve{g}_2(ẹ_3)=\{\text{¢}_2\}\cap\{\text{¢}_2\}=\{\text{¢}_2\}.$

Moreover,

$\gamma_3(ẹ_1,ẹ_2)=\gamma_1(ẹ_1)\cap\gamma_2(ẹ_2)=\{\text{¢}_3\}\cap\{\text{¢}_4\}=\emptyset,$
$\gamma_3(ẹ_1,ẹ_3)=\gamma_1(ẹ_1)\cap\gamma_2(ẹ_3)=\{\text{¢}_3\}\cap\{\text{¢}_3\}=\{\text{¢}_3\},$
$\gamma_3(ẹ_2,ẹ_2)=\gamma_1(ẹ_2)\cap\gamma_2(ẹ_2)=\{\text{¢}_3,\text{¢}_4\}\cap\{\text{¢}_4\}=\{\text{¢}_4\},$
$\gamma_3(ẹ_2,ẹ_3)=\gamma_1(ẹ_2)\cap\gamma_2(ẹ_3)=\{\text{¢}_3,\text{¢}_4\}\cap\{\text{¢}_3\}=\{\text{¢}_3\}.$

Also

$\delta_3(ẹ_1,ẹ_2)=\delta_1(ẹ_1)\cup\delta_2(ẹ_2)=\emptyset\cup\{\text{¢}_3,\text{¢}_4\}=\{\text{¢}_3,\text{¢}_4\},$
$\delta_3(ẹ_1,ẹ_3)=\delta_1(ẹ_1)\cup\delta_2(ẹ_3)=\emptyset\cup\{\text{¢}_4\}=\{\text{¢}_4\},$
$\delta_3(ẹ_2,ẹ_2)=\delta_1(ẹ_2)\cup\delta_2(ẹ_2)=\{\text{¢}_4\}\cup\{\text{¢}_3,\text{¢}_4\}=\{\text{¢}_3,\text{¢}_4\},$
$\delta_3(ẹ_2,ẹ_3)=\delta_1(ẹ_2)\cup\delta_2(ẹ_3)=\{\text{¢}_4\}\cup\{\text{¢}_4\}=\{\text{¢}_4\}.$

Definition 10: For two double framed T-bipolar soft sets $\left\{\left((\dot{f}_1,\breve{g}_1);\ \overline{\swarrow}\right),\left((\gamma_1,\delta_1);\ \overline{\swarrow}\right)\right\}$ and $\langle\left((\dot{f}_2,\breve{g}_2);B\right),\left((\gamma_2,\delta_2);B\right)\rangle$ over universal set $\underset{\smile}{U}$. The idea of "extended union" is denoted and defined by

$$\langle\left((\dot{f}_1,\breve{g}_1);\ \overline{\swarrow}\right),\left((\gamma_1,\delta_1);\ \overline{\swarrow}\right)\rangle\cup_{ẹxt.}\langle\left((\dot{f}_2,\breve{g}_2);B\right),\left((\gamma_2,\delta_2);B\right)\rangle=\begin{Bmatrix}\left((\dot{f}_1\cup\dot{f}_2,\breve{g}_1\cap\breve{g}_2);\ \overline{\swarrow}\cup B\right),\\ \left((\gamma_1\cap\gamma_2,\delta_1\cup\delta_2);\ \overline{\swarrow}\cup B\right)\end{Bmatrix} \quad (10)$$

Where

$\dot{f}_1\cup\dot{f}_2:\ \overline{\swarrow}\cup B\to P(X),\breve{g}_1\cap\breve{g}_2:\ \overline{\swarrow}\cup B\to P(X),\gamma_1\cap\gamma_2:\ \overline{\swarrow}\cup B\to P(Y)\ and\ \delta_1\cup\delta_2:\ \overline{\swarrow}\cup B\to P(Y)$ and $\dot{f}_1\cup\dot{f}_2:\ \overline{\swarrow}\cup B\to P(X)$ is defined by

$$\dot{f}_1 \cup \dot{f}_2(\underset{\wedge}{e}) = \begin{cases} \dot{f}_1(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \nearrow \backslash B \\ \dot{f}_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in B \backslash \nearrow \\ \dot{f}_1(\underset{\wedge}{e}) \cup \dot{f}_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \nearrow \cap B \end{cases} \quad (11)$$

$\breve{g}_1 \cap \breve{g}_2 : \nearrow \cup B \to P(X)$ is defined by

$$\breve{g}_1 \cap \breve{g}_2(\underset{\wedge}{e}) = \begin{cases} \breve{g}_1(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \nearrow \backslash B \\ \breve{g}_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in B \backslash \nearrow \\ \breve{g}_1(\underset{\wedge}{e}) \cap \breve{g}_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \nearrow \cap B \end{cases} \quad (12)$$

$\gamma_1 \cap \gamma_2 : \nearrow \cup B \to P(Y)$ is defined by

$$\gamma_1 \cap \gamma_2(\underset{\wedge}{e}) = \begin{cases} \gamma_1(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \nearrow \backslash B \\ \gamma_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in B \backslash \nearrow \\ \gamma_1(\underset{\wedge}{e}) \cap \gamma_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \nearrow \cap B \end{cases} \quad (13)$$

$\delta_1 \cup \delta_2 : \nearrow \cup B \to P(Y)$ is defined by

$$\delta_1 \cup \delta_2(\underset{\wedge}{e}) = \begin{cases} \delta_1(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \nearrow \backslash B \\ \delta_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in B \backslash \nearrow \\ \delta_1(\underset{\wedge}{e}) \cup \delta_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \nearrow \cap B \end{cases} \quad (14)$$

Definition 11: For two double framed T-bipolar soft sets $\left\{\left((\dot{f}_1, \breve{g}_1); \nearrow\right), \left((\gamma_1, \delta_1); \nearrow\right)\right\}$ and $\langle\left((\dot{f}_2, \breve{g}_2); B\right), \left((\gamma_2, \delta_2); B\right)\rangle$ over universal set $\underset{\sim}{U}$. The idea of a "restricted union" is denoted and defined by

$$\langle\left((\dot{f}_1, \breve{g}_1); \nearrow\right), \left((\gamma_1, \delta_1); \nearrow\right)\rangle \cup_{res.} \langle\left((\dot{f}_2, \breve{g}_2); B\right), \left((\gamma_2, \delta_2); B\right)\rangle$$
$$= \langle\left((\dot{f}_3, \breve{g}_3); C = \nearrow \cap B\right), \left((\gamma_3, \delta_3); C = \nearrow \cap B\right)\rangle \quad (15)$$

Where

$\dot{f}_3(\underset{\wedge}{e}) = \dot{f}_1(\underset{\wedge}{e}) \cup \dot{f}_2(\underset{\wedge}{e})$ and $\breve{g}_3(\underset{\wedge}{e}) = \breve{g}_1(\underset{\wedge}{e}) \cap \breve{g}_2(\underset{\wedge}{e})$ for all $\underset{\wedge}{e} \in C = \nearrow \cap B$. Also $\gamma_3(\underset{\wedge}{e}) = \gamma_1(\underset{\wedge}{e}) \cap \gamma_2(\underset{\wedge}{e})$ and $\delta_3(\underset{\wedge}{e}) = \delta_1(\underset{\wedge}{e}) \cup \delta_2(\underset{\wedge}{e})$ for all $\underset{\wedge}{e} \in C = \nearrow \cap B$.

Definition 12: For two double framed T-bipolar soft sets $\left\{\left((\dot{f}_1, \breve{g}_1); \nearrow\right), \left((\gamma_1, \delta_1); \nearrow\right)\right\}$ and $\langle\left((\dot{f}_2, \breve{g}_2); B\right), \left((\gamma_2, \delta_2); B\right)\rangle$ over universal set $\underset{\sim}{U}$. The idea of "restricted intersection" is denoted and defined by

$$\langle\left((\dot{f}_1, \breve{g}_1); \nearrow\right), \left((\gamma_1, \delta_1); \nearrow\right)\rangle \cap_{res.} \langle\left((\dot{f}_2, \breve{g}_2); B\right), \left((\gamma_2, \delta_2); B\right)\rangle$$
$$= \langle\left((\dot{f}_3, \breve{g}_3); C = \nearrow \cap B\right), \left((\gamma_3, \delta_3); C = \nearrow \cap B\right)\rangle \quad (16)$$

Where

$\dot{f}_3(\underset{\wedge}{e}) = \dot{f}_1(\underset{\wedge}{e}) \cap \dot{f}_2(\underset{\wedge}{e})$ and $\breve{g}_3(\underset{\wedge}{e}) = \breve{g}_1(\underset{\wedge}{e}) \cup \breve{g}_2(\underset{\wedge}{e})$ for all $\underset{\wedge}{e} \in C = \nearrow \cap B$. Also $\gamma_3(\underset{\wedge}{e}) = \gamma_1(\underset{\wedge}{e}) \cup \gamma_2(\underset{\wedge}{e})$ and $\delta_3(\underset{\wedge}{e}) = \delta_1(\underset{\wedge}{e}) \cap \delta_2(\underset{\wedge}{e})$ for all $\underset{\wedge}{e} \in C = \nearrow \cap B$.

Definition 13: For two double framed T-bipolar soft sets $\left\{\left((\dot{f}_1, \breve{g}_1); \nearrow\right), \left((\gamma_1, \delta_1); \nearrow\right)\right\}$ and $\langle\left((\dot{f}_2, \breve{g}_2); B\right), \left((\gamma_2, \delta_2); B\right)\rangle$ over universal set $\underset{\sim}{U}$. The idea of "extended intersection" is denoted and defined by

$$\langle\left((\dot{f}_1, \breve{g}_1); \nearrow\right), \left((\gamma_1, \delta_1); \nearrow\right)\rangle \cap_{ext.} \langle\left((\dot{f}_2, \breve{g}_2); B\right), \left((\gamma_2, \delta_2); B\right)\rangle = \begin{Bmatrix} \left((\dot{f}_1 \cap \dot{f}_2, \breve{g}_1 \cup \breve{g}_2); \nearrow \cup B\right), \\ \left((\gamma_1 \cup \gamma_2, \delta_1 \cap \delta_2); \nearrow \cup B\right) \end{Bmatrix} \quad (17)$$

Where

$\dot{f}_1 \cap \dot{f}_2 : \nearrow \cup B \to P(X), \breve{g}_1 \cup \breve{g}_2 : \nearrow \cup B \to P(X), \gamma_1 \cup \gamma_2 : \nearrow \cup B \to P(Y)\ and\ \delta_1 \cap \delta_2 : \nearrow \cup B \to P(Y)$ and $\dot{f}_1 \cap \dot{f}_2 : \nearrow \cup B \to P(X)$ is defined by

$$\dot{f}_1 \cap \dot{f}_2(\underset{\wedge}{e}) = \begin{cases} \dot{f}_1(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \nearrow \backslash B \\ \dot{f}_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in B \backslash \nearrow \\ \dot{f}_1(\underset{\wedge}{e}) \cap \dot{f}_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \nearrow \cap B \end{cases} \quad (18)$$

$\breve{g}_1 \cup \breve{g}_2 : \nearrow \cup B \to P(X)$ is defined by

$$\breve{g}_1 \cap \breve{g}_2(\underset{\wedge}{e}) = \begin{cases} \breve{g}_1(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \mathcal{A} \backslash B \\ \breve{g}_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in B \backslash \mathcal{A} \\ \breve{g}_1(\underset{\wedge}{e}) \cup \breve{g}_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \mathcal{A} \cap B \end{cases} \quad (19)$$

$\gamma_1 \cup \gamma_2 : \mathcal{A} \cup B \to P(Y)$ is defined by

$$\gamma_1 \cap \gamma_2(\underset{\wedge}{e}) = \begin{cases} \gamma_1(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \mathcal{A} \backslash B \\ \gamma_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in B \backslash \mathcal{A} \\ \gamma_1(\underset{\wedge}{e}) \cup \gamma_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \mathcal{A} \cap B \end{cases} \quad (20)$$

$\delta_1 \cap \delta_2 : \mathcal{A} \cup B \to P(Y)$ is defined by

$$\delta_1 \cup \delta_2(\underset{\wedge}{e}) = \begin{cases} \delta_1(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \mathcal{A} \backslash B \\ \delta_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in B \backslash \mathcal{A} \\ \delta_1(\underset{\wedge}{e}) \cap \delta_2(\underset{\wedge}{e}) & if\ \underset{\wedge}{e} \in \mathcal{A} \cap B \end{cases} \quad (21)$$

Definition 14: Let us consider the set of parameters by $\mathcal{A} = \{\underset{\wedge}{e}_1, \underset{\wedge}{e}_2, \underset{\wedge}{e}_3, \dots \underset{\wedge}{e}_l\} \subset E, X = \{x_1, x_2, x_3, \dots, x_m\}$ and $Y = \{y_1, y_2, y_3, \dots, y_n\}$ and $\langle\left((\dot{f}, \breve{g}); \mathcal{A}\right), \left((\gamma, \delta); \mathcal{A}\right)\rangle$ be the corresponding double framed T-bipolar soft set over $\underset{\sim}{U} = X \cup Y$. Then we can represent $\langle\left((\dot{f}, \breve{g}); \mathcal{A}\right), \left((\gamma, \delta); \mathcal{A}\right)\rangle$ as follows.

$$\mathfrak{f}_{ijk} = \begin{cases} (0,0,0,0)\ if\ x_j \notin \dot{f}(\underset{\wedge}{e}_i), x_j \notin \breve{g}(\underset{\wedge}{e}_i), y_k \notin \gamma(\underset{\wedge}{e}_i), y_k \notin \delta(\underset{\wedge}{e}_i) \\ (1,1,0,0)\ if\ x_j \in \dot{f}(\underset{\wedge}{e}_i), x_j \in \breve{g}(\underset{\wedge}{e}_i), y_k \notin \gamma(\underset{\wedge}{e}_i), y_k \notin \delta(\underset{\wedge}{e}_i) \\ (0,0,1,1)\ if\ x_j \notin \dot{f}(\underset{\wedge}{e}_i), x_j \notin \breve{g}(\underset{\wedge}{e}_i), y_k \in \gamma(\underset{\wedge}{e}_i), y_k \in \delta(\underset{\wedge}{e}_i) \\ (1,1,1,1)\ if\ x_j \in \dot{f}(\underset{\wedge}{e}_i), x_j \in \breve{g}(\underset{\wedge}{e}_i), y_k \in \gamma(\underset{\wedge}{e}_i), y_k \in \delta(\underset{\wedge}{e}_i) \end{cases} \quad (22)$$

Hence, a tabular representation of the data is given in Table 3.

Table 3. Representation of double framed T-bipolar soft sets

$\langle\left((\alpha,\beta);A\right),\left((\gamma,\delta);A\right)\rangle$	(x_1,y_1)	(x_1,y_2)	…	(x_1,y_n)	(x_2,y_1)	(x_2,y_2)	…	(x_2,y_n)	…
$\underset{\wedge}{e}_1$	$\mathfrak{f}_{111}$	$\mathfrak{f}_{112}$	…	$\mathfrak{f}_{11n}$	$\mathfrak{f}_{121}$	$\mathfrak{f}_{122}$	…	$\mathfrak{f}_{12n}$	…
$\underset{\wedge}{e}_2$	$\mathfrak{f}_{211}$	$\mathfrak{f}_{212}$	…	$\mathfrak{f}_{21n}$	$\mathfrak{f}_{221}$	$\mathfrak{f}_{222}$	…	$\mathfrak{f}_{22n}$	…
$\underset{\wedge}{e}_3$	$\mathfrak{f}_{311}$	$\mathfrak{f}_{312}$	…	$\mathfrak{f}_{31n}$	$\mathfrak{f}_{321}$	$\mathfrak{f}_{322}$	…	$\mathfrak{f}_{32n}$	…
…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…
$\underset{\wedge}{e}_l$	$\mathfrak{f}_{l11}$	$\mathfrak{f}_{l12}$	…	$\mathfrak{f}_{l1n}$	$\mathfrak{f}_{l21}$	$\mathfrak{f}_{l22}$	…	$\mathfrak{f}_{l2n}$	…

(x_m,y_1)	(x_m,y_2)	…	(x_m,y_n)
$\mathcal{F}_{1m1}$	$\mathfrak{f}_{1m2}$	…	$\mathfrak{f}_{1mn}$
$\mathcal{F}_{2m1}$	$\mathfrak{f}_{2m2}$	…	$\mathfrak{f}_{2mn}$
$\mathcal{F}_{3m1}$	$\mathfrak{f}_{3m2}$	…	$\mathfrak{f}_{3mn}$
…	…	…	…
…	…	…	…
$\mathcal{F}_{lm1}$	$\mathfrak{f}_{lm2}$	…	$\mathfrak{f}_{lmn}$

Now, based on these observations, we can easily take the data of double framed T-bipolar soft sets

5. Application of the proposed work

In this section, we present the application of the developed work.

5.1 Algorithm

Assume that $\mathcal{A} = \{\mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3, \dots, \mathbb{A}_l\}\ (i = 1,2,3,\dots,l)$ be the set of alternatives and $X = \{x_1, x_2, x_3, \dots, x_m\}, Y = \{y_1, y_2, y_3, \dots, y_n\}$ be the sets of attributes where $\underset{\sim}{U} = X \cup Y$. Now assume that the decision-makers provide their assessment in the form of double framed T-bipolar soft data based on the observation of equation (1). The step-wise algorithm is given by:

Step 1: Collect the information about each alternative based on attributes in the form of double framed T-bipolar soft sets and form a decision matrix as given in Table 4.

Table 4. Decision matrix

$\langle((\boldsymbol{\alpha},\boldsymbol{\beta});\boldsymbol{A}),((\boldsymbol{\gamma},\boldsymbol{\delta});\boldsymbol{A})\rangle$	$(\mathbb{x}_1,\mathbb{y}_1)$	$(\mathbb{x}_1,\mathbb{y}_2)$	…	$(\mathbb{x}_1,\mathbb{y}_n)$	$(\mathbb{x}_2,\mathbb{y}_1)$	$(\mathbb{x}_2,\mathbb{y}_2)$	…	$(\mathbb{x}_2,\mathbb{y}_n)$	…	$(\mathbb{x}_m,\mathbb{y}_1)$	$(\mathbb{x}_m,\mathbb{y}_2)$	…	$(\mathbb{x}_m,\mathbb{y}_n)$
$\mathbb{A}_1$	f_{111}	f_{112}	…	f_{11n}	f_{121}	f_{122}	…	f_{12n}	…	$\boldsymbol{\mathcal{F}}_{1m1}$	f_{1m2}	…	f_{1mn}
$\mathbb{A}_2$	f_{211}	f_{212}	…	f_{21n}	f_{221}	f_{222}	…	f_{22n}	…	$\boldsymbol{\mathcal{F}}_{2m1}$	f_{2m2}	…	f_{2mn}
$\mathbb{A}_3$	f_{311}	f_{312}	…	f_{31n}	f_{321}	f_{322}	…	f_{32n}	…	$\boldsymbol{\mathcal{F}}_{3m1}$	f_{3m2}	…	f_{3mn}
…	…	…	…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…	…
$\mathbb{A}_l$	f_{l11}	f_{l12}	…	f_{l1n}	f_{l21}	f_{l22}	…	f_{l2n}	…	$\boldsymbol{\mathcal{F}}_{lm1}$	f_{lm2}	…	f_{lmn}

Step 2: Find out the score values based on the formula developed as

$$\mathbb{S}_i = \overline{\mathcal{P}_i} - \underline{\mathcal{P}_i} \tag{23}$$

Where

$$\overline{\mathcal{P}_i} = \sum_{j,k} \mathbb{x}_{jk} \tag{24}$$

And

$$\underline{\mathcal{P}_i} = \sum_{j,k} \mathbb{y}_{jk} \tag{25}$$

Step 3: Find out $\max_i \mathbb{S}_i = \mathbb{S}_r$

Step 4: $\mathbb{S}_r$ is the best alternative.

5.2 Illustrative Example

Although DSP and MADM are two separate sciences, they can come together in a variety of uses, particularly when signal manipulation or analysis is used to aid in decision-making. Specific characteristics can be extracted from signals via DSP techniques. These characteristics could be used as MADM process attributes. DSP can be used to improve signals by eliminating irrelevant data or lowering noise. By doing this, the accuracy of the qualities employed in MADM is increased, improving the results of decision-making.

Table 5 provides potential details regarding each of the following four DSP approaches.

Convolution ($\mathbb{A}_1$): Convolution is an algorithm that shows how one signal changes another by combining two signals to create a third signal. Convolution is used in digital signal processing for sorting, which is the act of passing a source signal through a filter to produce an objective outcome, like recognizing edges or smoothness. A convolution technique is frequently used in relationships, processing sound, and picture editing. It is essential for linear model analysis.

Filtering ($\mathbb{A}_2$): A fundamental method in digital signal processing is filtering, which entails improving or changing specific features of a signal. Filters can be used to eliminate undesirable noise, extract relevant data, or modify the signal to the required shape. There are several kinds of filters, each with a distinct function, including band-pass, low-pass, high-pass, and notch filters. Fourier-based filters and FIR and IIR filters are examples of time-domain filters and frequency-domain filters, respectively. Filtering is used in signal demodulation in communication systems, image polishing, and noise elimination in audio transmissions.

Wavelet transform ($\mathbb{A}_3$): In DPS, the Wavelet Transform is an effective technique for analyzing signals at many different scales or frequencies. The Wavelet Transform is perfect for analyzing irregular signals since it provides combined time as well as frequency localization, unlike the Fourier Transform, which only provides frequency data. Wavelets are very helpful in areas like de-noising, which helps remove noise while maintaining crucial signal properties, and picture compression, which helps minimize file size without noticeably sacrificing quality. The Wavelet Transform is commonly utilized in geophysical sciences, finance, and medical imaging because it is flexible and can be applied to a variety of signal sources.

Fourier transform ($\mathbb{A}_4$): A key method in digital signal processing is the Fourier Transform, which is employed to translate a time-domain form input into its frequency-domain form. It breaks down the data into its levels so that its spectral parts can be examined. Commonly utilized DSP algorithms include the Discrete Fourier Transform (DFT) and Fast Fourier Transform (FFT), particularly for periodic signal analysis. FFT is a DFT variant that has been optimized for speedier computations.

Based on these alternatives, the set of attributes is given by

$\mathbb{x}_1$ =Parallel Processing, $\mathbb{x}_2$ =Low Latency, $\mathbb{x}_3$ =Low power consumption, $\mathbb{x}_4$ =High throughput

$\mathbb{y}_1$ =Flexible Architecture, $\mathbb{y}_2$ =Accuracy, $\mathbb{y}_3$ =Memory Bandwidth, $\mathbb{y}_4$ =Integration capability

Now, based on this defined information, assume that the decision-makers provide their assessment in the form of a double framed T-bipolar soft set $\langle\left((\dot{f},\breve{g});\barwedge\right),\left((\gamma,\delta);\barwedge\right)\rangle$ given by

$\dot{f}(\mathbb{A}_1)=\{\mathbb{x}_1,\mathbb{x}_2\},\dot{f}(\mathbb{A}_2)=\{\mathbb{x}_1\},\dot{f}(\mathbb{A}_3)=\{\mathbb{x}_1,\mathbb{x}_3\},\dot{f}(\mathbb{A}_4)=\{\mathbb{x}_2,\mathbb{x}_4\},$

$\breve{g}(\mathbb{A}_1)=\{\mathbb{x}_2,\mathbb{x}_4\},\breve{g}(\mathbb{A}_2)=\{\mathbb{x}_3\},\breve{g}(\mathbb{A}_3)=\{\mathbb{x}_1,\mathbb{x}_2\},\breve{g}(\mathbb{A}_4)=\{\mathbb{x}_1,\mathbb{x}_3\},$

$\gamma(\mathbb{A}_1)=\{\mathbb{y}_1\},\gamma(\mathbb{A}_2)=\{\mathbb{y}_1\},\gamma(\mathbb{A}_3)=\{\mathbb{y}_1\},\gamma(\mathbb{A}_4)=\{\mathbb{y}_1,\mathbb{y}_4\},$

$\delta(\mathbb{A}_1)=\{\mathbb{y}_2\},\delta(\mathbb{A}_2)=\{\mathbb{y}_2,\mathbb{y}_4\},\delta(\mathbb{A}_3)=\emptyset,\delta(\mathbb{A}_4)=\{\mathbb{y}_1,\mathbb{y}_3\}.$

Step 1: Based on the observation of equation (1), the tabular form of double framed T-bipolar soft set is given in Table 5.

Table 5. Double Framed T-bipolar soft data

$\langle\begin{matrix}((\alpha,\beta);A),\\((\gamma,\delta);A)\end{matrix}\rangle$	$\begin{pmatrix}\mathbb{x}_1,\\\mathbb{y}_1\end{pmatrix}$	$\begin{pmatrix}\mathbb{x}_1,\\\mathbb{y}_2\end{pmatrix}$	$\begin{pmatrix}\mathbb{x}_1,\\\mathbb{y}_3\end{pmatrix}$	$\begin{pmatrix}\mathbb{x}_1,\\\mathbb{y}_4\end{pmatrix}$	$\begin{pmatrix}\mathbb{x}_2,\\\mathbb{y}_1\end{pmatrix}$	$\begin{pmatrix}\mathbb{x}_2,\\\mathbb{y}_2\end{pmatrix}$	$\begin{pmatrix}\mathbb{x}_2,\\\mathbb{y}_3\end{pmatrix}$	$\begin{pmatrix}\mathbb{x}_2,\\\mathbb{y}_4\end{pmatrix}$	$\begin{pmatrix}\mathbb{x}_3,\\\mathbb{y}_1\end{pmatrix}$	$\begin{pmatrix}\mathbb{x}_3,\\\mathbb{y}_2\end{pmatrix}$
$\mathbb{A}_1$	$\begin{pmatrix}1,0,\\1,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,1\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}1,1,\\1,0\end{pmatrix}$	$\begin{pmatrix}1,1,\\0,1\end{pmatrix}$	$\begin{pmatrix}1,1,\\0,0\end{pmatrix}$	$\begin{pmatrix}1,1,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\1,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,1\end{pmatrix}$
$\mathbb{A}_2$	$\begin{pmatrix}1,0,\\1,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,1\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,1\end{pmatrix}$	$\begin{pmatrix}0,0,\\1,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,1\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,1\end{pmatrix}$	$\begin{pmatrix}0,1,\\1,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,1\end{pmatrix}$
$\mathbb{A}_3$	$\begin{pmatrix}1,1,\\1,0\end{pmatrix}$	$\begin{pmatrix}1,1,\\0,0\end{pmatrix}$	$\begin{pmatrix}1,1,\\0,0\end{pmatrix}$	$\begin{pmatrix}1,1,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\1,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\1,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,0\end{pmatrix}$
$\mathbb{A}_4$	$\begin{pmatrix}0,1,\\1,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,1\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,1\end{pmatrix}$	$\begin{pmatrix}0,1,\\1,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\1,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,1\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,1\end{pmatrix}$	$\begin{pmatrix}1,0,\\1,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\1,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,1\end{pmatrix}$

$(\mathbb{x}_3,\mathbb{y}_3)$	$(\mathbb{x}_3,\mathbb{y}_4)$	$(\mathbb{x}_4,\mathbb{y}_1)$	$(\mathbb{x}_4,\mathbb{y}_2)$	$(\mathbb{x}_4,\mathbb{y}_3)$	$(\mathbb{x}_4,\mathbb{y}_4)$
$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\1,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,1\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,0\end{pmatrix}$
$\begin{pmatrix}0,1,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,1,\\0,1\end{pmatrix}$	$\begin{pmatrix}0,0,\\1,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,1\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,1\end{pmatrix}$
$\begin{pmatrix}1,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\1,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$
$\begin{pmatrix}0,1,\\0,1\end{pmatrix}$	$\begin{pmatrix}0,1,\\1,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\1,0\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,1\end{pmatrix}$	$\begin{pmatrix}1,0,\\0,1\end{pmatrix}$	$\begin{pmatrix}1,0,\\1,0\end{pmatrix}$

Step 2: The overall results obtained by using the formulas (23)-(25) are given in Table 6.

Table 6. Overall result based on data given in Table 2

$\left\langle \begin{matrix}((\alpha,\beta);A),\\((\gamma,\delta);A)\end{matrix} \right\rangle$	$\overline{\mathcal{P}_i}$	$\underline{\mathcal{P}_i}$	$\mathbb{S}_i$
$\mathbb{A}_1$	16	8	8
$\mathbb{A}_2$	8	12	-4
$\mathbb{A}_3$	16	4	12
$\mathbb{A}_4$	16	16	0

Step 3: Find out $\max_i \mathbb{S}_i = \mathbb{S}_r$. In this case, we can see from Table 6 that $\mathbb{S}_3$ is the maximum value.

Step 4: Hence, according to the result given in Table 6, we can rank the alternative as $\mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_4 > \mathbb{A}_2$ and $\mathbb{A}_3$ is the best alternative.

6. Comparative analysis

This section is devoted to discussing the comparative analysis of the proposed work. We will describe how their proposed idea is more interesting and more advanced than other existing theories. We will compare our work with the idea of a soft set (Molodtsov, 1999), a T-bipolar soft set proposed by Mahmood (2020), and a double framed soft set (Jun and Ahn, 2012).

Note that

1. Although the idea of a double framed soft set is a more generalized notion than that of a soft set (Molodtsov, 1999), this notion is still limited because it can never discuss bipolarity. On the other hand, the developed idea can effectively discuss bipolarity. Moreover, if decision-makers provide their assessment in the form of a double framed soft set, then only a developed approach/algorithm can be used to handle such information. All other existing notions fail to handle double framed T-bipolar soft information. The graphical representation of the data given in Table 6 is given in Figure 3.

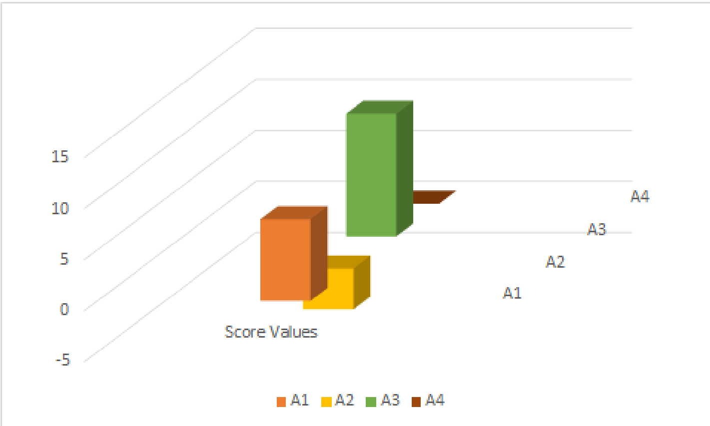

Figure 3. Graphical representation of data in Table 6

2. The notion of double framed soft set (Jun and Ahn, 2012) can never discuss the data given in Table 3, which is the imitation of this approach. The initiated notion can discuss all this information.

3. Mahmood's (2020) notion of T-bipolar soft structure is closer to bipolarity but these notions cannot discuss the information in the form of double framed T-bipolar soft sets. Hence we can say that if someone utilizes the notion of given in Mahmood (2020), then the chance of data loss increases. While the introduced approach can cover that kind of data loss and this characteristic makes it superior to other notions.

7. Conclusion

In many multi-criteria decision-making approaches, there is a need to utilize more advanced notions. To discuss the bipolarity in the structure, the notion of a T-bipolar soft set is more advanced. Based on these observations, in this manuscript, we have developed the idea of double framed T-bipolar soft sets. Moreover, we have defined the ideas of “AND” product, “OR” product, extended union, extended intersection, restricted union and restricted intersection for double framed T-bipolar soft sets. The product of two double framed T-bipolar soft sets has been developed. For the utilization of the introduced approach, we have initiated an algorithm for this purpose and provided an example to show the benefits and usefulness of the developed approach. The comparative analysis of the delivered work shows the superiority of the introduced work.

In the future, we can extend these notions to BCK/BCI algebra (Jun and Ahn, 2012), T-bipolar soft groups and rings theory as developed in Mahmood et al. (2024), and Ahmmad et al. (2024).

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